Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Aberrations: Non-Perfect Optical System
- Point source image defects:

- Ideal
- Coma
- Astigmatism
- Mixed
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Aberrations result from a Non-Perfect Optical System
  • *Definition of a perfect optical system:*
  • 1. Every ray or a pencil of rays proceeding from a single object point must, after passing through the optical system converge to a single point of the image. There can be no difference between chief and marginal rays intersection in the image plane! Ray trace of simple converging lens: ray 1 = marginal ; ray 2 = chief; and ray 3 = focal

![Diagram of ray paths through an optical system showing chief, marginal, and focal rays.]
Basic Optics : Microlithography
10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Definition of a perfect optical system:
  • 2. If the object is a plane surface perpendicular to the axis of the optical system, the image of any point on the object must also lie in a plane perpendicular to the axis. *This means that flat objects must be imaged as flat images and curved objects as curved images.*

• Image point is located at the common intersection of all rays which emanate from the corresponding object point
• The two rays passing through the two focal points and the chief ray can be ray-traced directly
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Definition of a perfect optical system:
  
  3. An image must be similar to the object whether it’s linear dimensions are altered or not. This means that irregular magnification or minification cannot occur in various parts of the image relative to the object. IDEAL CASE BELOW:

  PSF moves linearly and does not change shape

  ![Diagram](image)

  System is said to be **Space Invariant**, and

  \[ a_2 = -\frac{z_1}{z_0} a_0 = -Ma_0 \]

  Where \( M = \frac{z_1}{z_2} \) is the magnification of the system.

- Ray tracing using monochromatic light with image and **object located on the optical axis and paraxial rays (close to optic axis)** typically meet this perfect image criteria.
10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Aberrations results from:
  - **1. Defects due to nature (design):**
    - a. Dispersion: refractive index of glass varies with wavelength. (remember Shorter wavelength refracted more hence shorter focal length.)
    - b. Spherical space of lens surfaces
  - **2. Defects due to fabrication:** incorrect element spacing, tilted elements, rough glass surfaces, inhomogeneous glass (refractive index), glass stress, and incorrect element curvature or thickness.
  - **3. Defects due to application:** Thermal effects (lens heating), pressure changes, flare, and contamination (resist out-gassing or gas type change).
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Aberrations are defined as deviations in the image as a consequence of the light rays not obeying the 3 “perfect image” rules. The image location is not predicted by the classic ray trace procedure (applying Snells law at each surface). There is a wavefront deviation or optical path length difference (OPD) in the diffracted wavefronts forming the image.

• Aberration descriptions: mathematically described as wavefront deviations: for monochromatic aberrations.

• Zernike Polynomials: 37 terms from an infinite series; the magnitude of the coefficients (Z1 to Z37) values determine the aberrations in an optical system.

• Seidel Aberrations: “3rd” order approximation to Zernike terms.

• Basic aberrations include: Defocus (Z4), Astigmatism (Z5 Z6); Coma (Z7 Z8), Spherical (Z9) and field curvature and distortion.
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• **KEY ABERRATION IDEA:** OPD = Optical path length difference between the wavefront emerging from the lens aperture and the ideal reference wavefront. (I.e. Perfect non-aberrated wavefront)

• **Lens introduces a path length difference or phase shift!**

• *How this phase shift effects the image is the aberration!*
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Aberrations exist due to the idea that the wavefront emerging from the lens aperture (exit pupil) is deformed in shape and causes an OPD between emerging wave and reference wave.
• Aberrations are measured in wavelength OPD units.
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Aberrations exist due to the idea wavefront being deformed in shape and causing an OPD between emerging wave and reference wave.
• Aberrations are measured in wavelength OPD units
• *Lens produces a path length difference or phase shift*
• *Phase*

\[ \Phi = \kappa \Delta \quad \text{where} \quad \kappa = \frac{2\pi}{\lambda} \]

*Propagation constant in air*
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Phase function of lens:** $\Phi = \text{phase error}$

  with lens in place, at distance $h$ from optical,

  $$\Phi = \kappa \left( \delta_1 + \delta_2 + n(\Delta - \delta_1 - \delta_2) \right)$$

  which can be arranged to give

  $$\Phi = \kappa n \Delta - \kappa (n - 1)(\delta_1 + \delta_2)$$

  where $\delta_1$ and $\delta_2$ depend on $h$, the ray height.
Basic Optics : Microlithography
10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Optical aberrations
• **Monochromatic** : Field Curvature, spherical, astigmatism, coma, distortion
• **Polychromatic** : Chromatic and lateral color
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials
http://micro.magnet.fsu.edu/primer/anatomy/fieldcurvature.html

- **Monochromatic** : Field Curvature Aberration:
  Focus plane is curved.
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Monochromatic**: Field Curvature: Focus plane is curved. This natural curved focus plane is called Petzel surface.

  - *off-axis rays focus on a curved, not plane, surface*
Basic Optics: Microlithography
10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Field Curvature: This image defect is the **natural result** of using lenses that have curved surfaces. When visible light is focused through a curved lens, the image plane produced by the lens will be curved. A simple lens focuses image points from an extended flat object, such as a specimen on a microscope slide, onto a spherical surface resembling a curved bowl. **The nominal curvature of this surface is the reciprocal of the lens radius and is referred to as the Petzval Curvature of the lens.**

<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A - Y</th>
<th>Measurement</th>
<th>Microlithography Units specs modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Field Curvature</td>
<td>Petzval surface that is not flat plane</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Yes increases with the square of the image height Y</td>
<td>$Y^2$</td>
<td>Focal plane deviation across the field.</td>
<td>100nm</td>
<td>Spaced doublet; plan acrhomat</td>
</tr>
</tbody>
</table>
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials
http://micro.magnet.fsu.edu/primer/anatomy/fieldcurvature.html

- Field Curvature: Correction for flat field
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials
http://micro.magnet.fsu.edu/primer/anatomy/aberrationhome.html

- Spherical: The most serious of the monochromatic defects that occurs with objectives, spherical aberration, causes the image to appear hazy or blurred and slightly out of focus. The effect of spherical aberration manifests itself in two ways: the center remains more in focus than the edges of the image and the intensity of the edges falls relative to that of the center. This defect appears in both on-axis and off-axis image points. Simple explanation: lens is spherical (non parabolic) and outer rays focus “short”.

\[ \Delta z \]
\[ f \]
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials
http://micro.magnet.fsu.edu/primer/java/aberrations/spherical/index.html

• Spherical: measured as LSA and TSA
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Spherical:**

  Able to “cancel” some of the Spherical Aberration with defocus

  ![Spherical Aberration Diagram]

<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A - Y</th>
<th>Measurement</th>
<th>Microlithography Units spec modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Spherical</td>
<td>variation in focal plane with radial beam position</td>
<td>Monochromatic</td>
<td><strong>ON AXIS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bending, high index aspherics, gradient index, doublet</td>
</tr>
<tr>
<td></td>
<td>LSA</td>
<td>Increases as square of aperture diameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A^2$</td>
</tr>
<tr>
<td></td>
<td>TSA</td>
<td>Increases as cube of aperture diameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A^3$</td>
</tr>
</tbody>
</table>
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Astigmatism: vertical and horizontal objects have different foci due to lens curvature
• Astigmatism: vertical and horizontal objects have different foci due to lens curvature: Common fabrication error to make surfaces slightly cylindrical instead of perfectly spherical. In which case orthogonal wavefronts leaving the surface will have different radii.
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://micro.magnet.fsu.edu/primer/anatomy/aberrationhome.html

- **Astigmatism:** Complete removal of astigmatism is difficult, but can occur in optical systems when the two curves, $S$ and $T$, become flatter and coincide (Figure 3(c)), and the image is then formed in a region near the **Petzval surface** ($P$).

![Correction of Astigmatism](image)

Focal plane moving from “on-axis” = 0
**Basic Optics : Microlithography**

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Astigmatism:**

<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A - Y</th>
<th>Measurement</th>
<th>Microlithography Units spec modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Astigmatism</td>
<td>Rays in different planes do not have common foci</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Yes increases with the square of the image height Y</td>
<td>$Y^2$</td>
<td>V focus - H focus across the field</td>
<td>50 nm</td>
<td>Bending (spherical lens); Spaced doublet with stop</td>
</tr>
</tbody>
</table>
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Coma**: Also called off-axis spherical aberration; off axis singular object point imaging as multiple points

- **Tangential Coma of the lens** \( \text{Coma}_T = H''_{AB} - H'_p \)
10. Imaging Aberrations, Defocus, and Zernike Polynomials

• **Coma:** Comatic aberrations are similar to spherical aberrations, but they are mainly encountered with off-axis light fluxes and are most severe when the microscope is out of alignment. When these aberrations occur, the image of a point is focused at sequentially differing heights producing a series of asymmetrical spot shapes of increasing size that result in a comet-like (hence, the term coma; Figure 1) shape to the Airy pattern.
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Image with coma ($x_0$ plane):
Basic Optics : Microlithography
10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Coma can be corrected by using a combination of lenses that are positioned symmetrically around a central stop. In order to completely eliminate coma, the Abbe sine condition must be fulfilled:

\[ N_1 h_1 \sin \theta_1 = N_2 h_2 \sin \theta_2 \]

• The magnitude of Coma, like spherical aberration, is heavily dependent upon the shape of the lens. A strongly concave positive meniscus lens will demonstrate substantial negative comatic aberration, whereas plano-convex and bi-convex lenses produce comas that range from slightly negative to zero. Objects imaged through the convex side of a plano-convex lens or a convex meniscus lens will have a positive coma.

<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A Y</th>
<th>Measurement</th>
<th>Microlithography Units spec modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Coma</td>
<td>Variation in magnification with aperture (distance from optical axis)</td>
<td>Monochromatic</td>
<td>OFF AXIS Increases as square of aperture diameter</td>
<td>Yes increases linearly with the image height Y</td>
<td>A^2Y</td>
<td>left right linewidth deltas</td>
<td>50 nm</td>
<td>Bending ,spaced doublet with central stop</td>
</tr>
</tbody>
</table>

9/11/2004 Optics/Aberrations Steve Brainerd 26
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://micro.magnet.fsu.edu/primer/java/aberrations/distortion/index.html

- Distortion: off axis non-uniform magnification error across image field. Results in image placement errors.

No Distortion

$W_{311} > 0$
Barrel Distortion

$W_{311} < 0$
Pincushion Distortion
10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://micro.magnet.fsu.edu/primer/java/aberrations/distortion/index.html

• **Distortion**: The origin of geometrical distortion lies in a difference between the transverse magnification of a lens and the off-axis image distance. When this distance deviates from that predicted by paraxial theory for constant transverse magnification, distortion can arise due to differences in focal lengths and magnifications through various parts of the lens. In the absence of other aberrations, geometric distortion is manifested by a mis-shaped image, even though each image point is in sharp focus, as discussed above. Quantitatively, distortion can be described by the following equation:

\[ DM = (M_1 - M)/M \]

• where M is the axial lateral magnification and M(l) is the off-axis magnification at the image plane. If the lateral magnification increases proportionally with the off-axis distance of the object, distortion is positive, producing a pincushion effect (Figure 1). In this instance, each image point is displaced radially outward from the center, with the peripheral image points being displaced the greatest distance. Alternatively, when magnification is decreased with the off-axis object distance, distortion is negative and a barrel aberration is observed. Barrel distortion corresponds to a situation where the transverse magnification decreases with axial distance and each image point moves radially towards the center of the image.
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://micro.magnet.fsu.edu/primer/java/aberrations/distortion/index.html

- Distortion :
  - Correction:
  - Spaced doublet with central stop
  - Also corrects Coma and astigmatism

<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A - Y</th>
<th>Measurement</th>
<th>Microlithography Units spec modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Distortion</td>
<td>Radial displacement of off-axis points causing a field magnification error</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Yes increases with the cube of the image height Y</td>
<td>$Y^3$</td>
<td>Registration errors</td>
<td>Maximum error 20 nm</td>
<td>Spaced doublet with stop</td>
</tr>
</tbody>
</table>

9/11/2004  Optics/Aberrations  Steve Brainerd  29
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• **Polychromatic** : Chromatic (result of dispersion i.e. lens is like a prism)
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Polychromatic**: Lateral of traverse Chromatic aberration (Magnification changing with wavelength.)

![Diagram of Polychromatic aberration with rays for red and blue light, showing transverse chromatic aberration at the image plane.]

Aperture

Transverse Chromatic

Image Plane
**Basic Optics: Microlithography**

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Polychromatic**: Chromatic correction Achromatic lens
- **Doublet**

Power, Shape and index of each lens cancel the aberration of the other.

<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A - Y</th>
<th>Measurement</th>
<th>Microlithography Units spec modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Chromatic</td>
<td>on and off axis image blur</td>
<td>Polychromatic</td>
<td>ON AXIS Lateral OFF AXIS</td>
<td>red - blue focal plane</td>
<td>modern exposure tool bandwidth - 0.5 pm!</td>
<td>Achromatic lens (refractive index deltas to cancel dispersive powers)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Chromatic aberration effects:

Figure 1. Measurement of best focus as a function of central wavelength shows a linear relationship with a slope of 0.225 μm/pm for this 0.6 NA projection lens.
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Chromatic: Wavelength on exposure tools are not pure monochromatic: hence >> multiple wavelengths exist!

Figure 2. Examples of different KrF excimer laser spectra.
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Chromatic**: Focus dependence on wavelength: Using Zernike Z4 Focus (Note this reference uses calls it Z3.)


As an example, the response of wavelength as a focus shift can be modeled using the third fringe Zernike polynomial term (see reference 4 for a complete description of the Zernike polynomial used here). The coefficient of this Zernike term $Z_3$ can be related to a focus shift $\Delta \delta$ by

$$Z_3 = \Delta \delta \frac{NA^2}{4 \lambda_o} = (\text{slope})\Delta \lambda \frac{NA^2}{4 \lambda_o}$$  \hspace{1cm} (1)

where $\lambda_o$ is the central wavelength of the illumination spectrum. Thus, if the focus shift as a function of wavelength is known, a value of $Z_3$ for each wavelength in the illumination spectrum can be computed from the equation (1).
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• **Chromatic : Focus dependence on wavelength:**


![Graph showing aerial image intensity vs. horizontal position](image)

*Figure 3.* Degradation of the aerial image of a 180 nm line (500 nm pitch) with increasing laser bandwidth for a chromatic aberration response of 0.225 μm/pm.
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

- **Chromatic**: Process latitude dependence on excimer laser bandwidth wavelength:


![Graph](image_url)

**Figure 5.** Sensitivity of the focus-exposure process window to laser bandwidth. Numerical aperture of the lens is set at 0.6 and partial coherence factor $\sigma$ at 0.75.
<table>
<thead>
<tr>
<th>#</th>
<th>Aberration</th>
<th>Image Character</th>
<th>Wavelength</th>
<th>On axis and aperture A impact</th>
<th>Image Height Y off-axis Fct.</th>
<th>Total impact A - Y</th>
<th>Measurement</th>
<th>Microlithography Units spec modern Tool specs</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Field Curvature</td>
<td>Petzval surface that is not flat plane</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Yes increases with the square of the image height Y</td>
<td>Y^2</td>
<td>Focal plane deviation across the field.</td>
<td>0.2um</td>
<td>Spaced doublet; plan acrhomat</td>
</tr>
<tr>
<td>2</td>
<td>Spherical</td>
<td>variation in focal plane with radial beam position</td>
<td>Monochromatic</td>
<td>ON AXIS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bending , high index asphereics, gradient index, doublet</td>
</tr>
<tr>
<td>3</td>
<td>LSA</td>
<td>Increases as square of aperture diameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A^2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>TSA</td>
<td>Increases as cube of aperture diameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A^3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Astigmatism</td>
<td>Rays in different planes do not have common foci</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Yes increases with the square of the image height Y</td>
<td>Y^2</td>
<td>V focus - H focus across the field</td>
<td>0.10 um</td>
<td>Spaced doublet with stop</td>
</tr>
<tr>
<td>4</td>
<td>Coma</td>
<td>Variation in magnification with aperture (distance from optical axis)</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Increases linearly with the image height Y</td>
<td>A^2Y</td>
<td>left right linewidth deltas</td>
<td>0.05um</td>
<td>Bending , spaced doublet with central stop</td>
</tr>
<tr>
<td>5</td>
<td>Distortion</td>
<td>Radial displacement of off-axis points causing a field magnification error</td>
<td>Monochromatic</td>
<td>OFF AXIS</td>
<td>Yes increases with the cube of the image height Y</td>
<td>Y^3</td>
<td>Registration errors</td>
<td>Maximum error 20 nm</td>
<td>Spaced doublet with stop</td>
</tr>
<tr>
<td>6</td>
<td>Chromatic</td>
<td>on and off axis image blur</td>
<td>Polychromatic</td>
<td>ON AXIS</td>
<td></td>
<td></td>
<td>red - blue focal plane</td>
<td>modern exposure tool bandwidth - 0.5 pm</td>
<td>Achromatic lens (refractive index deltas to cancel dispersive powers)</td>
</tr>
</tbody>
</table>

**Basic Optics : Microlithography**

**Imaging Aberrations: Summary**
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Typical wavefront shapes at the entrance pupil for the primary aberrations. This can be observed in Prolith by inputting various aberrations
  • No Aberrations: wavefront shape
  • Can model in Prolith with Zernike input files lists magnitudes for the 37 terms
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

Prolith : coma wavefront errors Z7 Z8
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

Prolith: defocus wavefront errors Z4
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

Prolith : Spherical wavefront errors Z9
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

Prolith: multiple aberrations wavefront errors
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• The shape of these wavefronts is very complex. To analyze these and potentially correct the errors it is useful to fit the wavefront to a polynomial with each term (coefficient) representing a specific aberration. Two common Polynomials models (page 219 of text):

• **SEIDEL:** 1856 Philip Ludwig von Seidel in Germany (3rd order approximation to Zernike terms with power laws for variations across the pupil field.

• **Zernike Polynomials:** (can be cartesian (X,Y) or polar (R,q). They apply to a wavefront at a single point. To map the entire field the polynomial needs to be calculated at multiple points. Each term represents individually the best least squares fit of the data. This means to remove for example focus shift or coma from the wavefront, one just sets those terms to zero (Z4 for focus, Z8,Z9,Z14,Z15, Z23,Z24,Z34, and Z35 for XY coma). There are 37 terms. **Need to watch order and notation of coefficients when obtaining values!!**
Basic Optics: Microlithography
10. Imaging Aberrations, Defocus, and Zernike Polynomials

• SEIDEL's five aberrations
• The five (5) monochromatic aberrations analyzed by SEIDEL in Germany, in 1856:
  1.) spherical aberration
  2.) coma
  3.) astigmatism
  4.) curvature of field
  5.) distortion
Seidel Aberrations

- The derivations of the previous equations for the object, image and focal distances assumed $n_1 \theta_1 = n_2 \theta_2$. However, Snell's Law is exact; the inclusion of higher orders of the sine expansion produces slight alterations to the predictions of first-order theory. Adding the next non-zero term yields the approximation

$$\sin \theta = \theta - \frac{\theta^3}{3!},$$

which is used to define the third-order, or Seidel, aberrations. Aberrations refer to the difference in behavior of a real ray compared to an ideal ray. Wavefront error and transverse ray error are two methods of describing an aberration. Commonly used to describe the results of interferometric tests, the wavefront error compares the actual wavefront to a perfect, spherical wave converging to the focal point. Alternately, the transverse ray error is defined by the blur at the image plane for a specified object point and relates easily to geometric tests.
# Seidel Aberration Coefficients

<table>
<thead>
<tr>
<th>Wavefront Aberration Coefficient</th>
<th>Seidel Aberration Coefficient</th>
<th>Functional Form</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{200}$</td>
<td></td>
<td>$x_0^2$</td>
<td>piston</td>
</tr>
<tr>
<td>$W_{111}$</td>
<td></td>
<td>$x_0 \rho \cos \theta$</td>
<td>tilt</td>
</tr>
<tr>
<td>$W_{020}$</td>
<td></td>
<td>$\rho^2$</td>
<td>focus</td>
</tr>
<tr>
<td>$W_{040}$</td>
<td>$= \frac{1}{8} S_1$</td>
<td>$\rho^4$</td>
<td>spherical</td>
</tr>
<tr>
<td>$W_{131}$</td>
<td>$= \frac{1}{2} S_{11}$</td>
<td>$x_0 \rho^3 \cos \theta$</td>
<td>coma</td>
</tr>
<tr>
<td>$W_{222}$</td>
<td>$= \frac{1}{2} S_{111}$</td>
<td>$x_0^2 \rho^2 \cos^2 \theta$</td>
<td>astigmatism</td>
</tr>
<tr>
<td>$W_{220}$</td>
<td>$= \frac{1}{4} (S_{111} + S_{11v})$</td>
<td>$x_0^2 \rho^2$</td>
<td>field curvature</td>
</tr>
<tr>
<td>$W_{311}$</td>
<td>$= \frac{1}{2} S_v$</td>
<td>$x_0^3 \rho \cos \theta$</td>
<td>distortion</td>
</tr>
</tbody>
</table>
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

Seidel Aberrations : Seidel Wavefront error equation
\[ \Delta W = \text{wavefront error deviation from perfect wavefront} \]

In polar coordinates:
\[ x = \rho \cos \theta \quad \text{and} \quad y = \rho \sin \theta. \]

\[
W(x_0, \rho, \theta) = W_{200}x_0^2 + W_{111}x_0\rho \cos \theta + W_{020}\rho^2 \\
+ W_{040}\rho^4 + W_{131}x_0\rho^3 \cos \theta + W_{222}x_0^2\rho^2 \cos^2 \theta \\
+ W_{220}x_0^2\rho^2 + W_3x_0^3\rho \cos \theta;
\]

3rd order is the highest

\[ X_o = \text{image point} \]

Seidel aberration coefficients \( S_i \):
\[
W(x_0, \rho, \theta) = \frac{1}{8}S_1\rho^4 + \frac{1}{2}S_{\text{II}}x_0\rho^3 \cos \theta + \frac{1}{2}S_{\text{III}}x_0^2\rho^2 \cos^2 \theta \\
+ \frac{1}{4}(S_{\text{III}} + S_{\text{IV}})x_0^2\rho^2 + \frac{1}{2}S_{\text{V}}x_0^3\rho \cos \theta.
\]
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

Seidel Aberrations: Seidel Wavefront error equation
\[ \Delta W = \text{wavefront error deviation from perfect wavefront} \]

Seidel Aberrations
A. Spherical Aberration \( (\Delta W = W_{040}\rho^4 = W_{040}(x^2 + y^2)^2) \)
B. Coma \( (\Delta W = W_{131}x_0\rho^3\cos \theta = W_{131}x_0x(x^2 + y^2)) \)
C. Astigmatism \( (\Delta W = W_{222}x^2_0\rho^2\cos^2 \theta = W_{222}x^2_0x^2 \)
D. Field Curvature \( (\Delta W = W_{220}x^2_0\rho^2 = W_{220}x^2_0(x^2 + y^2)) \)
E. Distortion \( (\Delta W = W_{311}x^3_0\rho \cos \theta = W_{311}x^3_0x) \)
Basic Optics: Microlithography
10. Imaging Aberrations, Defocus, and Zernike Polynomials

T.A. Brunner.
Impact of Lens Aberrations on Optical Lithography.


- Imaging consequences of first 11 Zernike polynomials

*Note different Zernike terminology*

<table>
<thead>
<tr>
<th>(b, c)</th>
<th>Name</th>
<th>Imaging consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>Piston</td>
<td>None</td>
</tr>
<tr>
<td>(1, ± 1)</td>
<td>Lateral translation</td>
<td>Shift of image, independent of pattern</td>
</tr>
<tr>
<td>(2, 0)</td>
<td>Defocus</td>
<td>Image degradation</td>
</tr>
<tr>
<td>(2, ± 2)</td>
<td>Astigmatism (hor./vert. or ± 45°)</td>
<td>Orientation dependent shift of focus</td>
</tr>
<tr>
<td>(3, ± 1)</td>
<td>Lateral coma</td>
<td>Image asymmetry and pattern dependent shift of image</td>
</tr>
<tr>
<td>(3, ± 3)</td>
<td>Three-leaf clover (rotated 30°)</td>
<td>Imaging anomalies with threefold symmetry</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>Third-order spherical</td>
<td>Pattern dependent focus shift</td>
</tr>
</tbody>
</table>
Basic Optics: Microlithography

10. Imaging Aberrations

Phase error $W(p\theta)$ for terminology used here:

$$P(\rho,\theta) = P_i(\rho,\theta) \exp(i \, k \, W(\rho,\theta))$$  \hspace{1cm} k = \frac{2\pi}{\lambda}$$

$P(\rho,\theta)$ = generalized pupil function, $P_i(\rho,\theta)$ = ideal pupil function

$k \, W(\rho,\theta)$ = effective phase error \hspace{1cm} $W(\rho,\theta)$ = wave aberration

$$W(\rho,\theta) = a_{m,\rho,\theta=0} + a_1 \rho \cos \theta + a_2 \rho \sin \theta + a_3 \sqrt{3} (2\rho^2 - 1) + a_4 \sqrt{6} \rho^2 \sin 2\theta + a_5 \sqrt{6} \cos 2\theta + a_6 \sqrt{8} (3\rho^3 - 2) \sin \theta + a_7 \sqrt{8} (3\rho^3 - 2) \cos \theta + a_8 \sqrt{8} \rho^3 \sin 3\theta + a_9 \sqrt{8} \rho^3 \cos 3\theta + \ldots$$

where: \hspace{1cm} \rho = \sqrt{x^2 + y^2} \hspace{1cm} \theta = \tan^{-1} \frac{y}{x} \hspace{1cm} are pupil coordinates.

Basic Optics: Microlithography

10. Imaging Aberrations Fringe Zernike Polynomials

\[ \Phi_{n}(\rho, \theta) = c_{nm} \rho^{m} \sqrt{2(n + 1)} R_{n}^{m}(\rho) \cos m\theta, \]

which represents a term in the expansion of the aberration function in terms of a complete set of Zernike circle polynomials, which are orthonormal over a unit circular pupil, where \( n \) and \( m \) are positive integers (including zero), \( n - m \geq 0 \) and even. The radial polynomial

\[ R_{n}^{m}(\rho) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^{s} (n-s)!}{s![(n+m)/2-s]![n-m]/2-s]!} \rho^{n-2s}, \]

is a polynomial of degree \( n \) in \( \rho \) containing terms in \( \rho^{n}, \rho^{n-2}, \ldots, \rho^{m} \). The quantity

\[ \epsilon_{m} = 1/\sqrt{2}, m = 0 \]

\[ = 1, \quad m \neq 0. \]

Unless \( n = m = 0 \), the coefficient \( c_{nm} \) represents the standard deviation of the aberration across the pupil, i.e.,

\[ c_{nm} = \sigma_{\Phi}. \]

The orthonormality of the Zernike polynomials implies that

\[ \int_{0}^{2\pi} \int_{0}^{1} \Phi_{n}^{*}(\rho,\theta) \Phi_{m}^{*}(\rho,\theta) \rho d\rho d\theta \int_{0}^{1} \int_{0}^{2\pi} \rho d\rho d\theta = c_{nm}^{2} \delta_{nn} \delta_{mm}, \]

where \( \delta_{ij} \) is a Kronecker delta.

Ref: V. Mahajan, Aberration theory made simple, vol IT6, SPIE press.
Basic Optics : Microlithography

10. Imaging Aberrations Fringe Zernike Polynomials:

The common ordering of the Zernike coefficients is the “fringe” set from the University of Arizona. It is a subset of 37 terms from the infinite series.

Note some references start Z1 as X Tilt!

See page 224 of book Table 2 for the standard order. Fringe Zernike Polynomials:

Z1: Piston: Normalization term no effect (some references Z0)
Z2: X Tilt
Z3: Y Tilt
Z4: Defocus
Z5: 3\textsuperscript{rd} order Astigmatism 0 – 90\degree
Z6: 3\textsuperscript{rd} order Astigmatism 45 – 135\degree
Z7: X coma
Z8: Y coma
Z9: Spherical
Z10: 3-leaf Clover
Z11: 3-leaf Clover
Basic Optics: Microlithography
10. Imaging Aberrations Zernike Polynomials:

Zernike Polynomials: Another ordering method you might see:

<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>No.</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$\rho \cos \theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>$\rho \sin \theta'$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
<td>$2\rho^2 - 1$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>$\rho^2 \cos 2\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>$\rho^2 \sin 2\theta'$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td>$(3\rho^2 - 2)\rho \cos \theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>$(3\rho^2 - 2)\rho \sin \theta'$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>8</td>
<td>$6\rho^4 - 6\rho^2 + 1$</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>$\rho^2 \cos 3\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>$\rho^2 \sin 3\theta'$</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>11</td>
<td>$(4\rho^2 - 3)\rho^2 \cos 2\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>$(4\rho^2 - 3)\rho^2 \sin 2\theta'$</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>13</td>
<td>$(10\rho^4 - 12\rho^2 + 3)\rho \cos \theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>$(10\rho^4 - 12\rho^2 + 3)\rho \sin \theta'$</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>15</td>
<td>$20\rho^6 - 30\rho^4 + 12\rho^2 - 1$</td>
</tr>
</tbody>
</table>
**Basic Optics: Microlithography**

10. Imaging Aberrations Zernike Polynomials:

Zernike Polynomials: Another ordering method you might see:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m$</th>
<th>No.</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>16</td>
<td>$\rho^5 \cos 4\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17</td>
<td>$\rho^5 \sin 4\theta'$</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>18</td>
<td>$(5\rho^2 - 4)\rho^3 \cos 3\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>$(5\rho^2 - 4)\rho^3 \sin 3\theta'$</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>20</td>
<td>$(15\rho^4 - 20)\rho^2 + 6)\rho^2 \cos 2\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21</td>
<td>$(15\rho^4 - 72)\rho^2 + 6)\rho^2 \sin 2\theta'$</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>22</td>
<td>$(35\rho^6 - 60)\rho^3 + 30\rho^2 - 4)\rho \cos \theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23</td>
<td>$(35\rho^6 - 60)\rho^3 + 30\rho^2 - 4)\rho \sin \theta'$</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>24</td>
<td>$70\rho^8 - 140\rho^6 + 90\rho^4 - 20\rho^2 + 1$</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
<td>$\rho^5 \cos 5\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>26</td>
<td>$\rho^5 \sin 5\theta'$</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>27</td>
<td>$(6\rho^2 - 5)\rho^3 \cos 4\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28</td>
<td>$(6\rho^2 - 5)\rho^3 \sin 4\theta'$</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>29</td>
<td>$(21\rho^4 - 30)\rho^2 + 10)\rho^2 \cos 3\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>$(21\rho^4 - 30)\rho^2 + 10)\rho^2 \sin 3\theta'$</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>31</td>
<td>$(56\rho^6 - 105\rho^4 + 60\rho^2 - 10)\rho \cos 2\theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>32</td>
<td>$(56\rho^6 - 105\rho^4 + 60\rho^2 - 10)\rho \sin 2\theta'$</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>33</td>
<td>$(126\rho^8 - 280)\rho^6 + 210\rho^4 - 60\rho^2 + 5)\rho \cos \theta'$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34</td>
<td>$(126\rho^8 - 280)\rho^6 + 210\rho^4 - 60\rho^2 + 5)\rho \sin \theta'$</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>35</td>
<td>$252\rho^{10} - 630\rho^8 + 560\rho^6 - 210\rho^4 + 30\rho^2 - 1$</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>36</td>
<td>$924\rho^{12} - 2772\rho^{10} + 3150\rho^8 - 1680\rho^6 + 420\rho^4 - 42\rho^2 + 1$</td>
</tr>
</tbody>
</table>
Basic Optics: Microlithography

10. Imaging Aberrations Fringe Zernike Polynomials:

Simulations from Canon (Phil Ware)

- Z4 De-focus
- Z7 X coma
- Z8 Y coma
- Z2X-Tilt
- Z3 Y=Tilt
- Z5 3rd order 90° Astigmatism 0-90° (Δ V-H)
- Z6 3rd order 45° – 135° Astigmatism

0-theta: spherical aberration (smaller CD for iso than dense patterns, best focus deviation for different sizes AAPSM)
1-theta: coma aberration (CD difference between both end of 5 bar patterns)
2-theta: astigmatism (CD difference between two different orientations 0-90, 45-135)
3-theta: three-leaf clover (CD difference between 0-60-120, 30-90-150)
4-theta: 5 theta: higher order aberrations.
10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Testing lens for Wavefront errors: interferometer (Twyman Green): Siedel and Zernike mathematically describe these wavefront shapes = Quantified Aberration values!

Note the Double Pass through the lens under test.
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

• Testing lens for Wavefront errors: interferometer (Twyman Green):
  Interferograms:

  Interference between reference (flat) wavefront and double pass through test lens.

  Bright Fringe $\rightarrow$ OPD $= \pm n\lambda$
  Dark Fringe $\rightarrow$ OPD $= \pm (n + 1/2)\lambda$

  We get Contour Map of OPD. So Contour Map of

  $2W(u, v)$

  the wavefront aberration function.
10. Imaging Aberrations, Defocus, and Zernike Polynomials

- Testing lens for Wavefront errors: interferometer (Twyman Green):
  Interferograms:

Wavefronts with, $2\lambda$ of defocus, $2\lambda$ of defocus plus $3\lambda$ of tilt, and mixed aberrations.
Basic Optics : Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials
   http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

Wavefront Maps and Profiles for Seidel Aberrations

\[
\text{x-Tilt} = \frac{x}{x^2 + y^2}, \quad \text{y-Tilt} = \frac{y}{x^2 + y^2}, \quad \text{Focus} = \frac{x(x^2 + y^2)}{x^2 + y^2}, \quad \text{x-Coma} = \frac{x(x^2 + y^2)}{x^2 + y^2}, \quad \text{y-Coma} = \frac{y(x^2 + y^2)}{x^2 + y^2},
\]

Little of each

\[
\text{x-Astigmatism} = \frac{x^2}{(x^2 + y^2)^2}, \quad \text{y-Astigmatism} = \frac{y^2}{(x^2 + y^2)^2}, \quad \text{Spherical} = \frac{(x^2 + y^2)^2}{(x^2 + y^2)^2}
\]

\[
\text{Aberration} = 2x + 2x^3 + 2y + 2y^3 + 2(x^4 + y^4) + 2x(x^4 + y^4) + 2y(x^4 + y^4) + 2(x^4 + y^4)^4
\]
Wavefront Maps and Profiles for Seidel Aberrations

- x-Tilt $0 \times$, y-Tilt $0 \times$
- Focus $4 (x^2 + y^2)$
- x-Coma $2 x (x^2 + y^2)$
- y-Coma $0 y (x^2 + y^2)$
- x-Astigmatism $2 x^2$
- y-Astigmatism $2 y^2$
- Spherical $\frac{1}{2} (x^2 + y^2)^2$

Aberration $= 2 x^2 + 2 y^2 + 4 (x^2 + y^2) + 2 x (x^2 + y^2) + 2 (x^2 + y^2)^2$
Wavefront Maps and Profiles for Seidel Aberrations

- x-Tilt \[ \frac{0}{x} \]
- y-Tilt \[ \frac{0}{y} \]
- Focus \[ \frac{1}{(x^2 + y^2)} \]
- x-Coma \[ \frac{1}{x(x^2 + y^2)} \]
- y-Coma \[ \frac{1}{y(x^2 + y^2)} \]
- x-Astigmatism \[ \frac{1}{x^2} \]
- y-Astigmatism \[ \frac{1}{y^2} \]
- Spherical \[ \frac{1}{(x^2 + y^2)^2} \]

Aberration = \[ 2x^4 + 2y^4 + 4(x^2 + y^2) + 2x^2(x^2 + y^2) + 2y^2(x^2 + y^2) \]

9/11/2004  Optics/Aberrations  Steve Brainerd
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

- **Defocus: Z4**

\[ \text{Aberration} = -2(x^2 + y^2) \]
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

- Y Tilt : Z3

\[ \text{Aberration} = 2y \]
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

- 3rd order X Coma: $Z7$

$$\text{Aberration} = 2 \times (x^4 + y^4)$$
10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

- 3rd order Astigmatism: $Z_5$

$Aberration = 2x^2$
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

- 3rd order Astigmatism and 45 astigmatism: Z5 and Z6

\[
\begin{align*}
\text{x-Tilt } & 0 \, x, \quad \text{y-Tilt } 0 \, y, \quad \text{Focus } 0 \, (x^2+y^2), \\
\text{x-Coma } & 0 \, x(x^2+y^2), \quad \text{y-Coma } 0 \, y(x^2+y^2), \\
\text{x-Astigmatism } & 4 \, x^2, \quad \text{y-Astigmatism } 2 \, y^2, \quad \text{Spherical } 0 \, (x^2+y^2)^2
\end{align*}
\]

Aberration = \(4x^2 + 2y^2\)
Basic Optics: Microlithography

10. Imaging Aberrations, Defocus, and Zernike Polynomials

http://wyant.optics.arizona.edu/seidelWavefrontMaps/seidel.htm

- 3rd order Spherical: \( Z9 \)

\[
\text{Aberration} = 2 (x^2 + y^2)^2
\]