Multiple T-tests Vs AVOVA
Montgomery Chapter 3

$IV = \textit{independent variable}: DV \textit{dependent variable}$

- Suppose we have a data set as: We wish to see if the 3 means and the same or at least one is different.

<table>
<thead>
<tr>
<th>Replicate</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1005</td>
<td>995</td>
<td>950</td>
</tr>
<tr>
<td>2</td>
<td>1015</td>
<td>1008</td>
<td>975</td>
</tr>
<tr>
<td>3</td>
<td>1033</td>
<td>976</td>
<td>988</td>
</tr>
<tr>
<td>4</td>
<td>1028</td>
<td>1014</td>
<td>1015</td>
</tr>
<tr>
<td>5</td>
<td>1023</td>
<td>1011</td>
<td>1008</td>
</tr>
<tr>
<td>6</td>
<td>1043</td>
<td>982</td>
<td>994</td>
</tr>
</tbody>
</table>

$\bar{Y}_1 = 1024.50 \quad \bar{Y}_2 = 997.67 \quad \bar{Y}_3 = 988.33$

$S_1^2 = 179.90 \quad S_2^2 = 254.67 \quad S_3^2 = 554.47$
Multiple T-tests Vs ANOVA

• T-tests:

  • t-tests: 2 sample means and determined the probability that they were drawn from the same population.

• Analysis of Variance (ANOVA): Groups > 2

  • Like t-tests: ANOVA deals with differences between sample means.
  • Unlike t-tests it imposes no restriction on the number of means.
Multiple T-tests Vs ANOVA

Why not just use t-tests?

- Imagine you have 3 groups: Just use a series of independent groups t-test to test whether there are differences?

- 2 potential problems with this approach.

1. Too many comparisons!
   - There would be 3 different t tests:

   Group 1 v. Group 2
   Group 1 v. Group 3
   Group 2 v. Group 3
Multiple T-tests Vs AVOVA

- We could test all 3 means in a pair-wise comparison as:

\[
\begin{align*}
H_0: & \quad \mu_1 = \mu_2 \text{ vs. } H_a: \quad \mu_1 \neq \mu_2 \quad @ \alpha = 0.05 \\
H_0: & \quad \mu_1 = \mu_3 \text{ vs. } H_a: \quad \mu_1 \neq \mu_3 \quad @ \alpha = 0.05 \\
H_0: & \quad \mu_2 = \mu_3 \text{ vs. } H_a: \quad \mu_2 \neq \mu_3 \quad @ \alpha = 0.05
\end{align*}
\]

Each pairwise comparison is made at a type I error rate of 5%. This is known as the *comparisionwise* type I error rate.

Clearly, the chance of making at least one type I error across all comparison must be greater than or equal to 5%. This is known as the *experimentwise* type I error rate.
Multiple T-tests Vs ANOVA

- Number of t tests increases as the number of groups increases.
- Groups = 3, then we have 3 t tests
- 6 groups we would have to perform \(15\) different t tests.

Formula:

\[
C = \frac{(k)(k-1)}{2}
\]

- 4 groups \(C = \frac{4(4-1)}{2} = 6\)
Multiple T-tests Vs ANOVA

Pair-wise comparison $\alpha$ errors

Inflated probability of making a Type I error.
- Probability of at least one Type I error increases as the number of t tests increases.

- alpha at .05 indicates we are willing to risk being wrong 5% of the time when we reject the Ho.

- With many t-tests we are no longer just sampling 1 t-value from the t distribution.

- Result: alpha is no longer equal to .05. Probability of making a Type I error has gone. Experimentwise error
Multiple T-tests Vs AVOVA

• Pair-wise comparison $\alpha$ errors

If the comparisonwise type I error rate of 5% and the pairwise comparisons were independent then the experimentwise type I error would be: 5% for 2 treatments; 14% for three treatments; 26% for four treatments; and 40% for five treatments.

$$\text{Formula: } 1 - (1-\alpha)^C$$

4 groups: $C = 6$

$$1 - (1 - .05)^6 = .26$$
Multiple T-tests Vs AVOVA

Multiple t-test alpha risk error compounding:

<table>
<thead>
<tr>
<th>Number means to compare</th>
<th>Comparisons means taken at a time</th>
<th># T-tests COMBIN(#means,2)</th>
<th>$\alpha$ risk as $1-(1-\alpha)^{#tests}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5.00%</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>14.26%</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>6</td>
<td>26.49%</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>10</td>
<td>40.13%</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>15</td>
<td>53.67%</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>21</td>
<td>65.94%</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>28</td>
<td>76.22%</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>36</td>
<td>84.22%</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>45</td>
<td>90.06%</td>
</tr>
</tbody>
</table>

Alpha $\alpha$

0.05
Multiple T-tests Vs AVOVA

So, what’s the Advantage of an F-test?

- F-test: one overall comparison
- Avoids increased Type I error probability.

Definition of ANOVA

- Analysis of variance (ANOVA) is a statistical method for comparing 2 or more groups/set of observations.
- Same circumstances as the independent groups t-test except more levels of the IV.
- Relationship between t and F.
  - $t^2 = F$
  - Example: $t = 2.00$
  - F value? $F = 4.00$
Multiple T-tests Vs ANOVA

Null and Alternative?

- $H_0: \mu_1 = \mu_2 = \mu_3$
- $H_1: \mu_1 \neq \mu_2 \neq \mu_3$

What are we testing?
  - At least two of the means are different.

All means can be different or just one.

ANOVA just tells us that there is a difference!
Multiple T-tests Vs AVOVA
Montgomery Chapter 3

$IV = \text{independent variable}$: $DV = \text{dependent variable}$

**Testing Approaches - Analysis of Variance**

The term “analysis of variance” comes from the fact that this approach compares the variability observed among sample means to a pooled estimate of the variability among observations within each group.

Within group variance is small compared to variability among means.
Clear separation of means.

Within group variance is large compared to variability among means.
Unclear separation of means.
Multiple T-tests Vs AVOVA

Assumptions of the F-Test

1. **Homogeneity of Variance:** Each of our populations has the same variance. In other words:
   \[
   \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \ldots
   \]

2. **Normality:** The scores for each condition are normally distributed around the population mean.

3. **Independence of observations:** Observations within an experimental treatment are independent of one another.
Multiple T-tests Vs AVOVA

Violate Assumptions?

- ANOVA is a robust test
- Minimally affected by violations of population normality and homogeneity of variance so long as the samples are of equal size.
- **Big Problem:** violate homogeneity of variance and n-sizes are not equal
Multiple T-tests Vs AVOVA

**Logic of ANOVA-Hypothesis testing**

A quick review:

1. Assume that Ho is true.
2. State an expected result based on this assumption
3. Compute a sample statistic (t-test; F-test etc…).
4. Treat the statistic as a score in some known sampling distribution.
5. If the statistic falls within the rejection region, reject the null hypothesis.
6. Otherwise, fail to reject the null hypothesis

What’s new with ANOVA?
- Basic Logic does not change!
- Difference: use different statistical test.
  - F-test (test of the ratio between two variances)
Multiple T-tests Vs AVOVA

Distinguishing Two types of Variability

- Between-group variability and within group variability.

Within group versus Between group.

- Within group variability estimate: Individual differences, errors in measurement etc..
  - unexplained variability (sometimes called error variance).
Multiple T-tests Vs AVOVA

- Between group variability: Same sources as within, but also includes any variability due to group differences.
  - experimental manipulation

- Different ways to estimate variance basis of the F-test.

- Rather than using means to evaluate the Ho, the F-test is based on the ratio of the variances.

- ANOVA: calculate F

- F is the *ratio of 2 independent $\sigma^2$ estimates.*

  \[
  F = \frac{\text{variance b/n sample means}}{\text{variance expected by chance (error)}}
  \]

Remember these are estimates!!
Multiple T-tests Vs AVOVA

*IV = independent variable: DV dependent variable*

\[ F = \text{Differences among treatment means} \]
\[ \text{Differences among Ss treated alike} \]

If Ho is true:

\[ F = \text{Sampling error + Measurement error} = 1 \]
\[ \text{Sampling error + measurement error} \]

If Ho is false:

\[ F = \text{Sampling error + effect of IV + Measurement error} \]
\[ >1 \]
\[ \text{Sampling error + Measurement error} \]
Multiple T-tests Vs AVOVA

$IV = \text{independent variable; } DV = \text{dependent variable}$

EXCEL Example
Multiple T-tests Vs AVOVA

IV = independent variable; DV dependent variable

EXCEL Example

<table>
<thead>
<tr>
<th>Replicate</th>
<th>LAB 1</th>
<th>LAB 2</th>
<th>LAB 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1005</td>
<td>995</td>
<td>950</td>
<td></td>
</tr>
<tr>
<td>1015</td>
<td>1008</td>
<td>975</td>
<td></td>
</tr>
<tr>
<td>1033</td>
<td>976</td>
<td>988</td>
<td></td>
</tr>
<tr>
<td>1028</td>
<td>1014</td>
<td>1015</td>
<td></td>
</tr>
<tr>
<td>1023</td>
<td>1011</td>
<td>1008</td>
<td></td>
</tr>
<tr>
<td>1043</td>
<td>982</td>
<td>994</td>
<td></td>
</tr>
</tbody>
</table>
Multiple T-tests Vs AVOVA

*IV = independent variable: DV dependent variable*

**EXCEL Example**

<table>
<thead>
<tr>
<th>Replicate</th>
<th>LAB 1</th>
<th>LAB 2</th>
<th>LAB 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1005</td>
<td>995</td>
<td>950</td>
<td></td>
</tr>
<tr>
<td>1015</td>
<td>1008</td>
<td>975</td>
<td></td>
</tr>
<tr>
<td>1033</td>
<td>976</td>
<td>988</td>
<td></td>
</tr>
<tr>
<td>1028</td>
<td>1014</td>
<td>1015</td>
<td></td>
</tr>
<tr>
<td>1023</td>
<td>1011</td>
<td>1008</td>
<td></td>
</tr>
<tr>
<td>1043</td>
<td>982</td>
<td>994</td>
<td></td>
</tr>
</tbody>
</table>

**Anova: Single Factor**

<table>
<thead>
<tr>
<th>Groups</th>
<th>Count</th>
<th>Sum</th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAB 1</td>
<td>6</td>
<td>6147</td>
<td>1025</td>
<td>180</td>
</tr>
<tr>
<td>LAB 2</td>
<td>6</td>
<td>5986</td>
<td>998</td>
<td>255</td>
</tr>
<tr>
<td>LAB 3</td>
<td>6</td>
<td>5930</td>
<td>988</td>
<td>555</td>
</tr>
</tbody>
</table>

**ANOVA**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>4230.333</td>
<td>2</td>
<td>2115.166667</td>
<td>6.40938</td>
<td>0.01</td>
<td>3.68</td>
</tr>
<tr>
<td>Within Groups</td>
<td>4950.167</td>
<td>15</td>
<td>330.0111111</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>9180.5</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**CONCLUSION:** Labs are different! We will use this same type of test to compare FACTOR effects in an experiment!