Design of Engineering Experiments
Chapter 4 – The Blocking Principle

• Text Reference, Chapter 4
• **Blocking** and **nuisance factors**
• The randomized complete block design or the **RCBD**
• Extension of the ANOVA to the RCBD
• Other blocking scenarios...Latin square designs (Note Could not get JUMP or Design Expert to do A Latin Square Design Directly.)
The Blocking Principle

- **Blocking** is a technique for dealing with **nuisance factors**
- A **nuisance** factor is a factor that probably has some affect on the response, but it’s of no interest to the experimenter…however, the variability it transmits to the response needs to be minimized
- Typical nuisance factors include batches of raw material, operators, pieces of test equipment, time (shifts, days, etc.), different experimental units
- **Many** industrial experiments involve blocking (or should)
- Failure to block is a common flaw in designing an experiment (consequences?)
The Blocking Principle

- If the nuisance variable is **known** and **controllable**, we use **blocking**
- If the nuisance factor is **known** and **uncontrollable**, sometimes we can use the **analysis of covariance** (see Chapter 14) to remove the effect of the nuisance factor from the analysis
- If the nuisance factor is **unknown** and **uncontrollable** (a "lurking" variable), we hope that **randomization** balances out its impact across the experiment
- Sometimes several sources of variability are **combined** in a block, so the block becomes an aggregate variable
The Hardness Testing Example

- Text reference, pg 127
- We wish to determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
- Gauge & measurement systems capability studies are frequent areas for applying DOX
- Assignment of the tips to an experimental unit; that is, a test coupon
- Structure of a completely randomized experiment
- The test coupons are a source of nuisance variability
- Alternatively, the experimenter may want to test the tips across coupons of various hardness levels
- The need for blocking
The Hardness Testing Example

- To conduct this experiment as a RCBD, assign all 4 tips to each coupon.
- Each coupon is called a “block”; that is, it’s a more homogenous experimental unit on which to test the tips.
- Variability between blocks can be large, variability within a block should be relatively small.
- In general, a block is a specific level of the nuisance factor.
- A complete replicate of the basic experiment is conducted in each block.
- A block represents a restriction on randomization.
- All runs within a block are randomized.
The Hardness Testing Example

• Suppose that we use: \( a = 4 \) treatments; \( b = 4 \) blocks:

<table>
<thead>
<tr>
<th>Type of Tip</th>
<th>Test Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>9.3</td>
</tr>
<tr>
<td>2</td>
<td>9.4</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
</tr>
<tr>
<td>4</td>
<td>9.7</td>
</tr>
</tbody>
</table>

• Notice the two-way structure of the experiment
• Once again, we are interested in testing the equality of treatment means (effect of the tips), but now we have to remove the variability associated with the nuisance factor (the blocks)
Extension of the ANOVA to the RCBD

• Suppose that there are $a$ treatments (factor levels) and $b$ blocks

• A statistical model (effects model) for the RCBD is

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{cases}$$

• The relevant (fixed effects) hypotheses are

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a \text{ where } \mu_i = (1/b) \sum_{j=1}^{b} (\mu + \tau_i + \beta_j) = \mu + \tau_i$$
Extension of the ANOVA to the RCBD

ANOVA partitioning of total variability:

\[\sum_{i=1}^{a} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{a} \sum_{j=1}^{b} [(\bar{y}_{i.} - \bar{y}_{..}) + (\bar{y}_{.j} - \bar{y}_{..})] \]

\[+ (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2\]

\[= b \sum_{i=1}^{a} (\bar{y}_{i.} - \bar{y}_{..})^2 + a \sum_{j=1}^{b} (\bar{y}_{.j} - \bar{y}_{..})^2\]

\[+ \sum_{i=1}^{a} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2\]

\[SS_T = SS_{Treatments} + SS_{Blocks} + SS_E\]
Extension of the ANOVA to the RCBD

The degrees of freedom for the sums of squares in
\[ SS_T = SS_{\text{Treatments}} + SS_{\text{Blocks}} + SS_E \]
are as follows:
\[ ab - 1 = a - 1 + b - 1 + (a - 1)(b - 1) \]

Therefore, ratios of sums of squares to their degrees of freedom result in mean squares and the ratio of the mean square for treatments to the error mean square is an \( F \) statistic that can be used to test the hypothesis of equal treatment means.
**ANOVA Display for the RCBD**

Table 4-2  Analysis of Variance for a Randomized Complete Block Design

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>( F_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>( SS_{\text{Treatments}} )</td>
<td>( a - 1 )</td>
<td>( \frac{SS_{\text{Treatments}}}{a - 1} )</td>
<td>( MS_{\text{Treatments}} )</td>
</tr>
<tr>
<td>Blocks</td>
<td>( SS_{\text{Blocks}} )</td>
<td>( b - 1 )</td>
<td>( \frac{SS_{\text{Blocks}}}{b - 1} )</td>
<td>( MS_{E} )</td>
</tr>
<tr>
<td>Error</td>
<td>( SS_E )</td>
<td>( (a - 1)(b - 1) )</td>
<td>( \frac{SS_E}{(a - 1)(b - 1)} )</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>( SS_T )</td>
<td>( N - 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Manual computing (ugh!)…see Equations (4-9) – (4-12), page 131

Design-Expert or Jump analyzes the RCBD
ANOVA RCBD Design Expert: Tip measurement of hardness example
Are all tips the same? Blocked on coupons:

<table>
<thead>
<tr>
<th>Std</th>
<th>Run</th>
<th>Block</th>
<th>Factor 1 A:Hardness Tip Type</th>
<th>Response 1 Hardness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>Coupon 1</td>
<td>1</td>
<td>9.3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>Coupon 2</td>
<td>1</td>
<td>9.4</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>Coupon 3</td>
<td>1</td>
<td>9.6</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>Coupon 4</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>Coupon 1</td>
<td>2</td>
<td>9.4</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>Coupon 2</td>
<td>2</td>
<td>9.3</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>Coupon 3</td>
<td>2</td>
<td>9.8</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>Coupon 4</td>
<td>2</td>
<td>9.9</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>Coupon 1</td>
<td>3</td>
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</tr>
<tr>
<td>10</td>
<td>3</td>
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<td>3</td>
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<tr>
<td>11</td>
<td>9</td>
<td>Coupon 3</td>
<td>3</td>
<td>9.5</td>
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<tr>
<td>12</td>
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<td>Coupon 4</td>
<td>3</td>
<td>9.7</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>Coupon 1</td>
<td>4</td>
<td>9.7</td>
</tr>
<tr>
<td>14</td>
<td>16</td>
<td>Coupon 2</td>
<td>4</td>
<td>9.6</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>Coupon 3</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>Coupon 4</td>
<td>4</td>
<td>10.2</td>
</tr>
</tbody>
</table>
ANOVA RCBD Design Expert:
Tip measurement of hardness example
Are all tips the same? Blocked on coupons:

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<tr>
<td>2</td>
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<td>9.4</td>
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<tr>
<td>3</td>
<td>13</td>
<td></td>
<td>1</td>
<td>9.6</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td></td>
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<td>10</td>
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<tr>
<td>5</td>
<td>11</td>
<td></td>
<td>2</td>
<td>9.4</td>
</tr>
<tr>
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<td>10</td>
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<td>2</td>
<td>9.3</td>
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<tr>
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<td>9.8</td>
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<td></td>
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<td>9.2</td>
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<tr>
<td>10</td>
<td>3</td>
<td></td>
<td>3</td>
<td>9.4</td>
</tr>
<tr>
<td>11</td>
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<td>14</td>
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<td>4</td>
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<tr>
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<td>16</td>
<td></td>
<td>4</td>
<td>9.6</td>
</tr>
<tr>
<td>15</td>
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<td>4</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td></td>
<td>4</td>
<td>10.2</td>
</tr>
</tbody>
</table>

ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>0.82</td>
<td>3</td>
<td>0.27</td>
<td>14.44</td>
<td>0.0009</td>
</tr>
<tr>
<td>Model</td>
<td>0.38</td>
<td>3</td>
<td>0.13</td>
<td>14.44</td>
<td>0.0009</td>
</tr>
<tr>
<td>Residual</td>
<td>0.080</td>
<td>9</td>
<td>8.889E-003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>1.29</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No the tips are different!
### ANOVA RCBD Design Expert: Tip measurement of hardness example

Coded data Same result

<table>
<thead>
<tr>
<th>Tip</th>
<th>Hardness</th>
<th>Hardness CODED = ( Y - 9.5)*10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.3</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>9.4</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>9.6</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>9.4</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>9.3</td>
<td>-2</td>
</tr>
<tr>
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<td>3</td>
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<td>9.9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>9.4</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>9.5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>9.7</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>9.7</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>9.6</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>10.2</td>
<td>7</td>
</tr>
</tbody>
</table>

### Analysis of variance table [Partial sum of squares]

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>82.50</td>
<td>3</td>
<td>27.50</td>
<td>14.44</td>
<td>0.0009</td>
</tr>
<tr>
<td>Model</td>
<td>38.50</td>
<td>3</td>
<td>12.83</td>
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</tr>
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<td>A</td>
<td>38.50</td>
<td>3</td>
<td>12.83</td>
<td>14.44</td>
<td>0.0009</td>
</tr>
<tr>
<td>Residual</td>
<td>8.00</td>
<td>9</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>129.00</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Model Fit Statistics

- Std. Dev.: 0.94
- R-Squared: 0.8280
- Mean: 1.25
- Adj R-Squared: 0.7706
- C.V.: 75.42
- Pred R-Squared: 0.4563
- PRESS: 25.28
- Adeq Precision: 15.635
Hardness Testing Revisited (pg. 132)

The textbook analysis is conducted in terms of **coded data**

**Table 4-4  Coded Data for the Hardness Testing Experiment**

<table>
<thead>
<tr>
<th>Type of Tip</th>
<th>Coupon (Block)</th>
<th></th>
<th></th>
<th></th>
<th>( y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2</td>
<td>-1</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

\( y_{ij} \) = \( -4 \) \( -3 \) \( 9 \) \( 18 \) \( 20 = y_{..} \)
# Hardness Testing Revisited

**Design-Expert Output**

**Response:** Hardness

### ANOVA for Selected Factorial Model

**Analysis of variance table [Partial sum of squares]**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>82.50</td>
<td>3</td>
<td>27.50</td>
<td>14.44</td>
<td>0.0009</td>
</tr>
<tr>
<td>Model</td>
<td>38.50</td>
<td>3</td>
<td>12.83</td>
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<tr>
<td>A</td>
<td>38.50</td>
<td>3</td>
<td>12.83</td>
<td>14.44</td>
<td>0.0009</td>
</tr>
<tr>
<td>Residual</td>
<td>8.00</td>
<td>9</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>129.00</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Std. Dev. 0.94
- Mean 1.25
- C.V. 75.42
- PRESS 25.28

- R-Squared 0.8280
- Adj R-Squared 0.7706
- Pred R-Squared 0.4563
- Adeq Precision 15.635
Residual Analysis for the Hardness Testing Experiment

![Residual Analysis Plot]

Blocking Montgomery Chapter 4
Steve Brainerd
Residual Analysis for the Hardness Testing Experiment
Residual Analysis for the Hardness Testing Experiment

- Basic residual plots indicate that normality, constant variance assumptions are satisfied
- No obvious problems with randomization
- Can also plot residuals versus the type of tip (residuals by factor) and versus the blocks...these plots are in Figure 4-5, pg. 137 of the text
- These plots provide more information about the constant variance assumption, possible outliers
### Multiple Comparisons for the Hardness Testing Experiment – *Which Tips are Different?*

**Treatment Means (Adjusted, If Necessary)**

<table>
<thead>
<tr>
<th></th>
<th>Estimated Mean</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-T1</td>
<td>0.75</td>
<td>0.47</td>
</tr>
<tr>
<td>2-T2</td>
<td>1.00</td>
<td>0.47</td>
</tr>
<tr>
<td>3-T3</td>
<td>-0.50</td>
<td>0.47</td>
</tr>
<tr>
<td>4-T4</td>
<td>3.75</td>
<td>0.47</td>
</tr>
</tbody>
</table>

| Treatment Difference | Mean Difference | DF | Standard Error | t for $H_0$ Coeff=0 | Prob > |t| |
|----------------------|-----------------|----|----------------|----------------------|--------|--------|
| 1 vs 2               | -0.25           | 1  | 0.67           | -0.38                | 0.7163 |
| 1 vs 3               | 1.25            | 1  | 0.67           | 1.87                 | 0.0935 |
| **1 vs 4**           | -3.00           | 1  | 0.67           | -4.50                | **0.0015** |
| 2 vs 3               | 1.50            | 1  | 0.67           | 2.25                 | 0.0510 |
| **2 vs 4**           | -2.75           | 1  | 0.67           | -4.13                | **0.0026** |
| **3 vs 4**           | -4.25           | 1  | 0.67           | -6.38                | **0.0001** |

*Also see Figure 4-3, Pg. 135*
Multiple Comparisons for the Hardness Testing Experiment –

Tip 4 is significantly different from 1, 2, and 3!

Also see Figure 4-3, Pg. 135
Other Aspects of the RCBD
See Text, Section 4-1.3, pg. 136

• The RCBD utilizes an **additive model** – no interaction between treatments and blocks
• Treatments and/or blocks as random effects
• Missing values
• What are the **consequences of not blocking** if we should have? See pages 132-133 and Table 4-6: *We would have stated no difference in the tips, when there was!!*
## Blocking vs not Blocking

- If we had **not blocked** if we should have? We would have an inflated error term causing an inability to detect a difference in the treatments (tips):

### Data Table

<table>
<thead>
<tr>
<th>Std</th>
<th>Run</th>
<th>Block</th>
<th>Factor 1 A:Hardness Tip Type</th>
<th>Response 1 Hardness CODED</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>none</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>none</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>none</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
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<td>5</td>
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<tr>
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<td>9</td>
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<td>11</td>
<td>none</td>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>none</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>none</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>none</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>none</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>none</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>none</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
<td>none</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

### ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>Value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>38.50</td>
<td>3</td>
<td>12.83</td>
<td>1.70</td>
<td>0.2196</td>
</tr>
<tr>
<td>Pure Error</td>
<td>90.50</td>
<td>12</td>
<td>7.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>129.00</td>
<td>15</td>
<td>7.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.75</td>
<td></td>
<td>R-Squared</td>
<td>0.2984</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.25</td>
<td></td>
<td>Adj. R-Squared</td>
<td>0.1231</td>
<td></td>
</tr>
<tr>
<td>C.V.</td>
<td>219.70</td>
<td></td>
<td>Pred R-Squared</td>
<td>-0.2472</td>
<td></td>
</tr>
<tr>
<td>PRESS</td>
<td>160.89</td>
<td></td>
<td>Adeq Precision</td>
<td>3.095</td>
<td></td>
</tr>
</tbody>
</table>

**Not Blocked: Tips are the same!!**
The Latin Square Design

- Text reference, Section 4-2, pg. 144
- These designs are used to simultaneously control (or eliminate) **two sources of nuisance variability**
- A significant assumption is that the three factors (treatments, nuisance factors) **do not interact**
- If this assumption is violated, the Latin square design will not produce valid results
- Latin squares are not used as much as the RCBD in industrial experimentation.
- Latin square is for equal groups of \( p \) factors or a \( p \times p \) square see designs on page 145!
The Rocket Propellant Burn Rate Problem – A Latin Square Design

- **Problem:** We are interested in seeing if there is a difference in five rocket propellant formulations effect on burn rate, we define as A,B,C,D,E. We know that there are two Nuisance type factors: **Batches of chemicals** and **Operators** who mix the materials. Since these are know and controllable we can setup our experimental design to “factor” them out using a “Latin Square design” as shown below.

<table>
<thead>
<tr>
<th>Batches of Raw Material</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4-9 Latin Square Design for the Rocket Propellant Problem**

<table>
<thead>
<tr>
<th>Operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
The Rocket Propellant Problem – A Latin Square Design

Table 4-9 Latin Square Design for the Rocket Propellant Problem

<table>
<thead>
<tr>
<th>Batches of Raw Material</th>
<th>Operators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>A = 24</td>
</tr>
<tr>
<td>2</td>
<td>B = 17</td>
</tr>
<tr>
<td>3</td>
<td>C = 18</td>
</tr>
<tr>
<td>4</td>
<td>D = 26</td>
</tr>
<tr>
<td>5</td>
<td>E = 22</td>
</tr>
</tbody>
</table>

- This is a 5×5 Latin square design
- Page 145 shows some other Latin squares
- Table 4-13 (page 148) contains properties of Latin squares
- Statistical analysis?
Statistical Analysis of the Latin Square Design

• The statistical (effects) model is

\[ y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \]

\[
\begin{align*}
  &i = 1, 2, \ldots, p \\
  &j = 1, 2, \ldots, p \\
  &k = 1, 2, \ldots, p 
\end{align*}
\]

• The statistical analysis (ANOVA) is much like the analysis for the RCBD.
• See the ANOVA table, page 146 (Table 4-10)
• The analysis for the rocket propellant example is presented on text pages 146 & 147
Statistical Analysis of the Latin Square Design; Cannot do a Latin Square on Jump or Design Expert Block on Batches of Raw Material

<table>
<thead>
<tr>
<th>Operator</th>
<th>Raw Material</th>
<th>Propellant Formulation</th>
<th>Burn rate ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>B</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>C</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>D</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>E</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>B</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>C</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>D</td>
<td>38</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>E</td>
<td>31</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>A</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>C</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>D</td>
<td>30</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>E</td>
<td>26</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>A</td>
<td>26</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>B</td>
<td>20</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>D</td>
<td>24</td>
</tr>
<tr>
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<td>4</td>
<td>E</td>
<td>27</td>
</tr>
<tr>
<td>18</td>
<td>4</td>
<td>A</td>
<td>27</td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td>B</td>
<td>23</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>C</td>
<td>29</td>
</tr>
<tr>
<td>21</td>
<td>5</td>
<td>E</td>
<td>24</td>
</tr>
<tr>
<td>22</td>
<td>5</td>
<td>A</td>
<td>36</td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>B</td>
<td>21</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>C</td>
<td>22</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
<td>D</td>
<td>31</td>
</tr>
</tbody>
</table>
Statistical Analysis of the Latin Square Design; Cannot do a Latin Square on Jump or Design Expert Block on Batches of Raw Material
Statistical Analysis of the
Latin Square Design; Cannot do a Latin Square on Jump or Design
Expert: Block on Operators
**Statistical Analysis of the Latin Square Design Combine results from both ANOVA blocked on each factor to complete Latin Square analysis in EXCEL (manual)**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Obtained from:</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ANOVA Latin Square Rocket Propellant</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formulations</td>
<td>From Jump</td>
<td>330</td>
<td>4</td>
<td>82.5</td>
<td>7.73438</td>
<td>0.002536502</td>
</tr>
<tr>
<td>Batches of Raw Material</td>
<td>Jump blocked on Raw Materials</td>
<td>68</td>
<td>4</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operators</td>
<td>Jump blocked on Operators</td>
<td>150</td>
<td>4</td>
<td>37.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>Total -( Sum For, B and O)</td>
<td>128</td>
<td>12</td>
<td>10.6667</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>Jump</td>
<td>676</td>
<td>24</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>