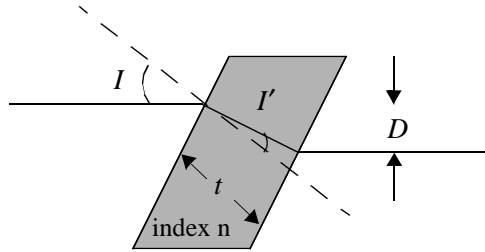


Some Prism Devices

[Reading assignment: Jenkins and White, Sections 2.2 (review), 2.3. See also Smith Ch. 4]

Lateral displacement of a ray passing obliquely through a plane parallel glass plate:



$$D = t \sin I \left[ 1 - \sqrt{\frac{1 - \sin^2 I}{n^2 - \sin^2 I}} \right]$$

for small incidence angle  $I \rightarrow i$

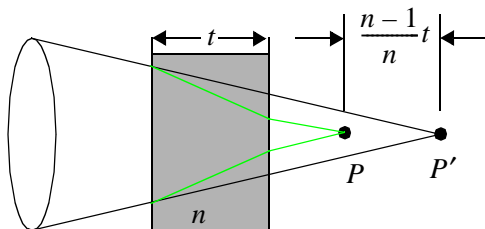
$$D = ti \frac{n-1}{n}$$

This can be used to laterally displace an image. One application of this very simple device is in a specialized high speed camera. The film has to move so fast that it is driven continuously (rather than actually stopping briefly for each frame as in a conventional camera). A rotating plate is used to make the image track the moving film during exposure of a given frame to prevent blur.

But the plate introduces aberrations.

- Chromatic effect: longitudinal and lateral displacements depend on  $n$  which is  $\lambda$  dependent.
- For a plate used in convergant or divergent light, the amount of displacement is greater for larger angles which gives spherical aberration.

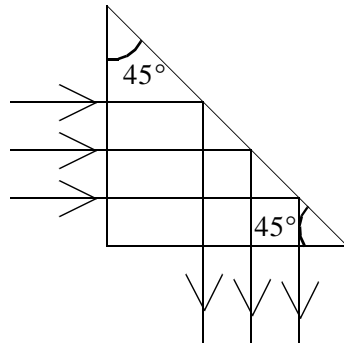
Plane parallel plate placed in between a lens and its focus:



A simple calculation based on the paraxial approximation shows that the focus is displaced by amount

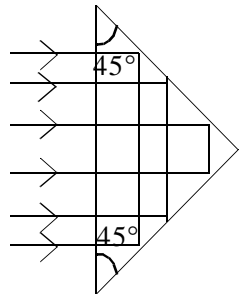
$\frac{n-1}{n}t$ . However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration.

**Right Angle Prism**



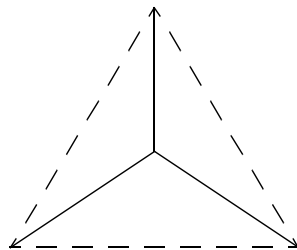
common building block in non-dispersive prism devices

**Porro prism**



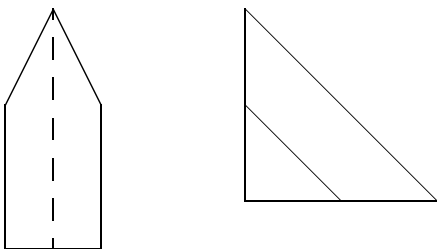
retro reflector (only folds back on itself in one meridian)

**Corner cube**



(beam reflects back on itself regardless of incident direction)

**Roof Prism**

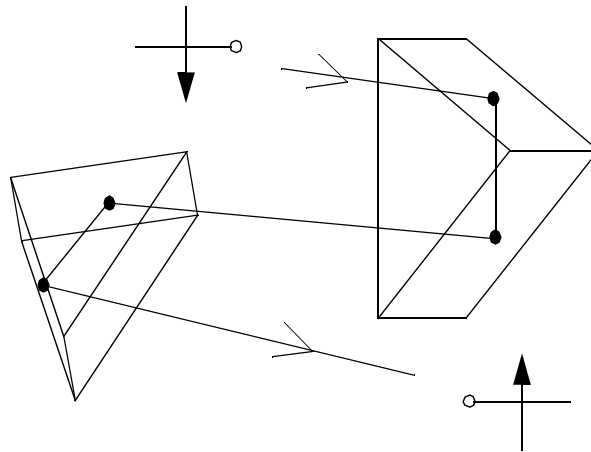


right angle prism with a roof on hypotenuse

provides an inversion of L-R which right angle prism does not.

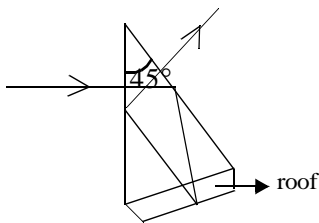
### Erecting Prisms

Most telescopes produce an inverted image (both U-D, L-R) to the eye. Erecting prisms re-invert the image to the proper orientation.

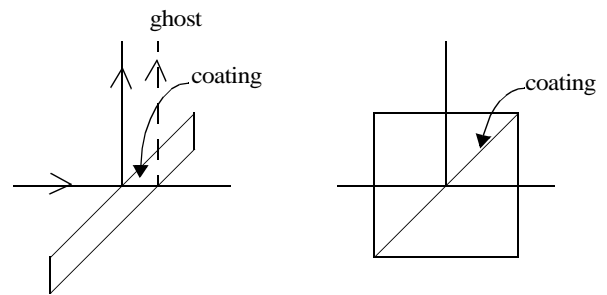


2 porro prisms used together.  
Generally contacted

### Schmidt Prism

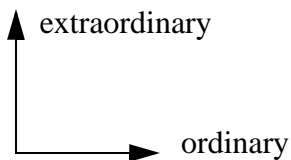


### Prism Beamsplitter



### Polarizing Prisms

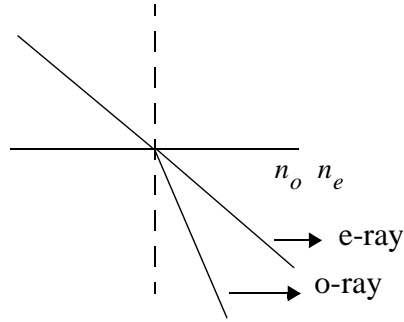
Birefringent crystals: given a propagation direction in the crystal - a set of orthogonal axes can be determined. For the two polarization directions along these axes, the index is different.



The ordinary component has index  $n_o$  and the extraordinary component has index  $n_e$ . As propagation direction varies the ordinary component always has index  $n_o$  but  $n_e$  varies between  $n_o$  and  $n_{e0}$ .

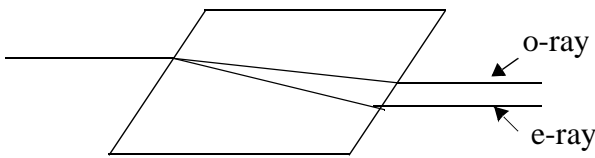
### Double Refraction

Light incident on a birefringent crystal



o-ray pol and e-ray pol are not necessarily same as s and p pol.

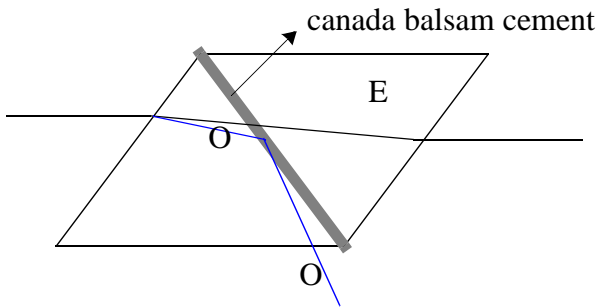
**Birefringent plate**



Common birefringent crystals-quartz ( $\text{SiO}_2$ ), calcite ( $\text{CaCO}_3$ )

Not birefringent: Si, NaCl, GaAs, diamond

**Nicol Prism**



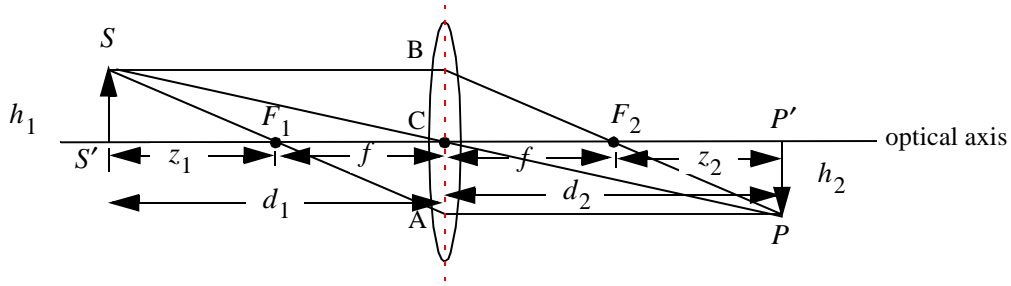
Calcite  $n_o = 1.658$   
 $n_e = 1.486$   
 $n_B = 1.55$

O-ray is internally reflected at the calcite-balsam interface  $n_B > n_o$  e-ray is transmitted (angle is cut for Brewster)

Optical Imaging Systems

[Reading assignment: Jenkins and White, Sections 3.1-3.7, 3.10, Chapter 4]

Thin lens, focal length  $f$



- Rays entering the lens parallel to the axis, pass through the back focus,  $F_2$
- Rays passing through the front focus,  $F_1$  are “collimated” and emerge parallel to the axis.
- Rays passing through the center of the lens  $C$  are not bent.
- For a “thin lens”, the rays are all bent at the lens plane, with no translation.
- Sign conventions: Heights above the axis are positive, below the axis are negative; left is negative, right is positive. Focal length for converging lens is positive; diverging lens is negative.

$$\boxed{h_1, f, z_2, d_2} \text{ are positive}$$

$$z_1, d_1, h_2 \text{ are negative}$$

Triangles  $S'SF_1$  and  $ACF_1$  are similar, so

$$\boxed{\frac{h_1}{-z_1} = \frac{-h_2}{f}}$$

Triangles  $BCF_2$  and  $PP'F_2$  are similar, so

$$\boxed{\frac{h_1}{f} = \frac{-h_2}{z_2}}$$

These two equations give us:  $\frac{z_1}{f} = \frac{-f}{z_2}$  or  $\boxed{z_1 z_2 = -f^2}$  “Newtonian” form of lens law

Now use  $z_1 = d_1 + f$  ,  $z_2 = d_2 - f$  (watch signs!)

$$f^2 = -z_1 z_2 = -(d_1 + f)(d_2 - f)$$

$$f^2 = -d_1 d_2 - f d_2 + d_1 f + f^2$$

cancel  $f^2$  , divide by  $d_1 d_2 f$

## LENSES AND IMAGING

$$\boxed{\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1}}$$

“Gaussian” lens law

The lateral or transverse image magnification:

$$m \equiv h_2/h_1 = f/z_1 = \frac{f}{d_1 + f}$$

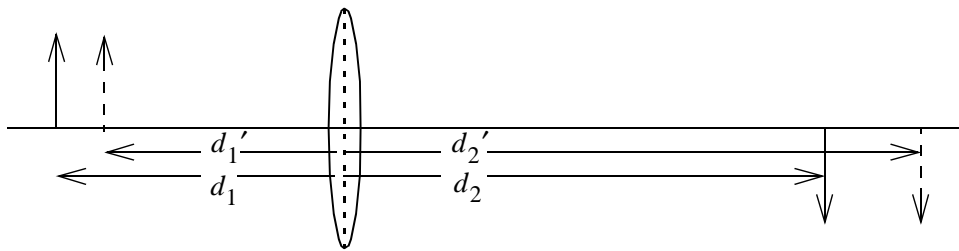
Use the lens law to get  $d_2 = \frac{d_1 f}{d_1 + f}$ .

so we also find  $\boxed{m = d_2/d_1}$

The longitudinal magnification is of interest.

For a given small shift of an object along the optic axis, how much does the image shift?

We define the longitudinal magnification as:



$$\bar{m} \equiv \frac{\partial d_2}{\partial d_1} = \frac{(d_1 + f)f - d_1 f}{(d_1 + f)^2} = \boxed{\frac{f_2}{(d_1 + f)^2} = m^2}$$

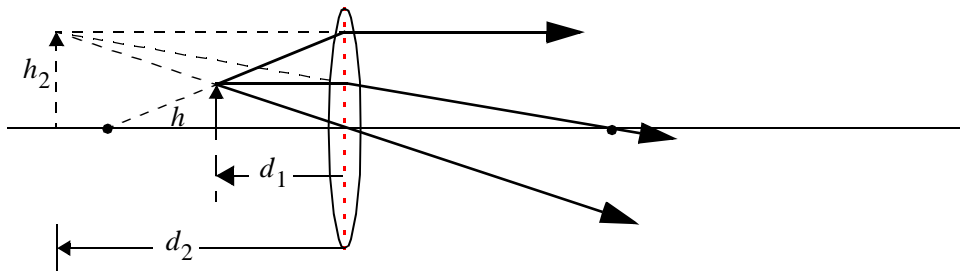
The longitudinal magnification is positive and the square of the transverse magnification

### Virtual Image

For an object to the left of the lens,  $d_1$  is negative. Since

$$\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1}$$

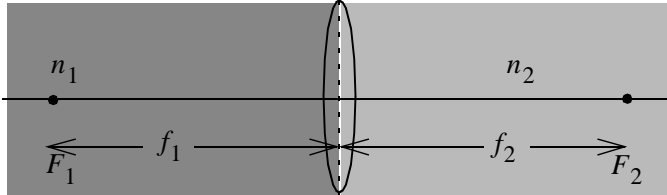
Then if  $|d_1| < f$ , then  $d_2$  is also negative.



## LENSES AND IMAGING

The light emerging from the lens appears to be coming from the object with height  $h_1$  at distance  $d_2$  behind the lens.

For an optical system not immersed in air



The front and back focal lengths are not the same in this case

front focal length:  $f_1$

back focal length:  $f_2$

$$\frac{f_1}{n_1} = \frac{f_2}{n_2}$$

lens law becomes:

$$\frac{n_2}{d_2} = \frac{n_1}{d_1} + \frac{n_1}{f_1} = \frac{n_1}{d_1} + \frac{n_2}{f_2}$$

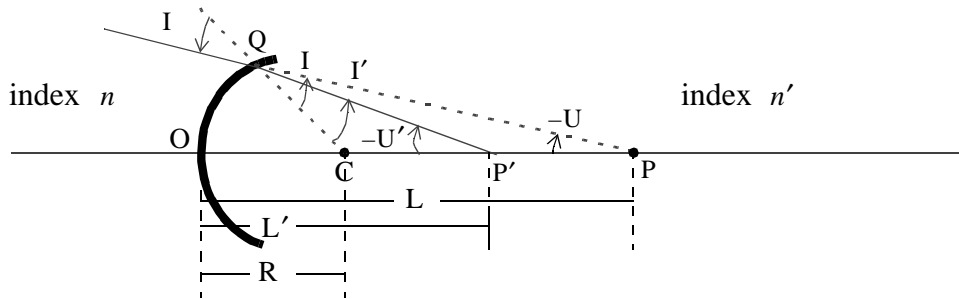
$$z_1 z_2 = -f_1 f_2$$

$$m = \frac{h_2}{h_1} = \frac{n_1 d_2}{n_2 d_1}$$

$$\bar{m} = \frac{f_1 f_2}{z_1}$$

$$\bar{m} \neq m^2 \quad (\text{show } \bar{m} = \frac{n_2}{n_1} m^2)$$

### Refraction of light by a spherical surface (following Smith 2.4)



#### Sign Conventions

1. Radius is positive when center of curvature is to the right of the surface
2. Distance to the right of surface is positive; left negative.
3. For  $I, I'$ , counterclockwise from the surface normal is positive.
4. For  $U, U'$ , the angle is positive if the ray slope is positive.

5. Rays travel left to right

In the diagram above all the quantities are positive except  $U, U'$ .

Consider triangle  $QCP$ . By law of sines:

$$\boxed{\frac{\sin I}{L-R} = \frac{-\sin U}{R}} \quad \text{(Eq. 1)}$$

Similarly for triangle  $QCP'$ :

$$\frac{\sin I'}{L'-R} = \frac{-\sin U'}{R} \quad \text{(Eq. 2)}$$

Comparing triangles  $QCP$  and  $QCP'$ , we see they share a common angle. Therefore

$$\boxed{I - U = I' - U'} \quad \text{(Eq. 3)}$$

Finally, by Snell's law

$$n \sin I = n' \sin I' \quad \text{(Eq. 4)}$$

We can arrive at the Gaussian lens law (for the single surface) from these equations.

Manipulate Eq. 1:

$$\begin{aligned} \frac{\sin I}{\sin U} &= \frac{R-L}{R} = 1 - \frac{L}{R} \\ \frac{L}{R} &= 1 - \frac{\sin I}{\sin U} = \frac{\sin U - \sin I}{\sin U} \\ \frac{R}{L} &= \frac{\sin U}{\sin U - \sin I} = 1 - \frac{\sin I}{\sin I - \sin U} \end{aligned}$$

Now multiply by  $\frac{n}{R}$  to get

$$\boxed{\frac{n}{L} = \frac{n}{R} - \frac{\sin I}{\sin I - \sin U}} \quad \text{(Eq. 5)}$$

Similarly, from Eq.2,

$$\frac{n'}{L'} = \frac{n'}{R} - \frac{n' \sin I'}{R \sin I' - \sin U'} \quad \text{(Eq. 6)}$$

Subtract Eq.5 from Eq.6

$$\frac{n'}{L'} - \frac{n}{L} = \frac{n' - n}{R} - \left[ \frac{n \sin I'}{R \sin I' - \sin U'} - \frac{n \sin I}{R \sin I - \sin U} \right]$$

Using Eq. 4,

$$\boxed{\frac{n'}{L'} - \frac{n}{L} = \frac{n' - n}{R} - \frac{n \sin I}{R} \left[ \frac{1}{\sin I' - \sin U'} - \frac{1}{\sin I - \sin U} \right]} \quad (\text{Eq. 7})$$

This has the form of the lens law, except for the dependence on the sin of all the angles.

We shall see that Gaussian Optics applies to spherical surfaces only in the Paraxial Approximation.

### Paraxial Approximation

The paraxial approximation refers to the case when all ray angles remain small. (Close to the optic axis.) In this case, for all angles,  $\sin x \cong x$   $\tan x \cong x$ . By convention, the lower case letter is substituted for the capital in this approximation, so

$$\sin I \rightarrow i, \sin I' \rightarrow i', \sin U \rightarrow u, \sin U' \rightarrow u'$$

$$L \rightarrow l, L' \rightarrow l'$$

R is unaffected

Eqs. 1-4 become

$$\boxed{\frac{i}{l - R} = \frac{-u}{R}} \quad (\text{Eq. 8})$$

$$\frac{i'}{l' - R} = \frac{-u'}{R} \quad (\text{Eq. 9})$$

$$i - u = i' - u' \quad (\text{Eq. 10})$$

$$\boxed{ni = n'i'} \quad (\text{Eq. 11})$$

Then Eq. 7 becomes:

$$\boxed{\frac{n'}{l'} - \frac{n}{l} = \frac{n' - n}{R}} \quad (\text{Eq. 12})$$

This is the Gaussian lens law for a single surface.

Consider a ray incident from the left, parallel to the axis. Then  $l \rightarrow -\infty$ , and we have

$$\frac{n'}{l'} = \frac{n' - n}{R}$$

$$\boxed{l' = \frac{n'}{n' - n} R}$$

Thus, the back focal length,  $f_2$ , is  $\frac{n'}{n' - n} R$ . Similarly, for an image distance of  $\infty$ , we must have  $l \rightarrow \infty$  and

$$\frac{-n}{l} = \frac{n' - n}{R}$$

$$l = -\frac{n}{n' - n} R.$$

This means the front focal length  $f_1$  is  $\boxed{\frac{n}{n' - n}R}$

(Watch minus sign!)

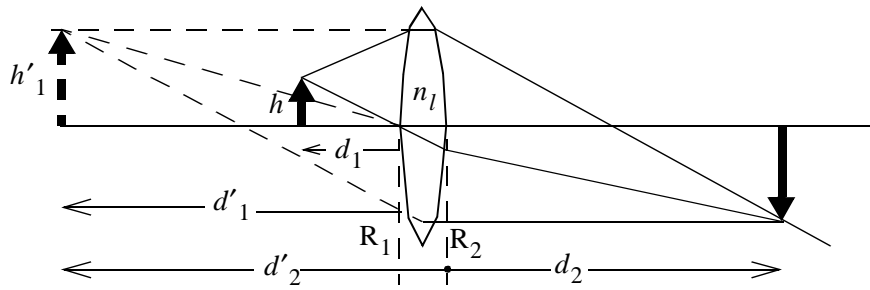
Recall the previous discussion for a lens not immersed in air:

$$\frac{f_1}{n_1} = \frac{f_2}{n_2} ; \frac{n_1}{d_2} = \frac{n_1}{d_1} + \frac{n_1}{f_1} = \frac{n_1}{d_1} + \frac{n_2}{f_2}$$

These relations clearly hold for the single spherical surface.

### Thin Lens Model

We now construct our model for the thin lens in air. Lens index is  $n_l$ . We will find the imaging properties of the thin lens by using the previous results for a single spherical surface and applying them twice - once for each of the two surfaces of the lens.



Use Eq. 12 for first surface:  $n = 1$   $n' = n_l$ ,  $l \rightarrow d_1$ ,  $l' \rightarrow d'_1$

$$\boxed{\frac{n_l}{d'_1} - \frac{1}{d_1} = \frac{n_l}{R_1}} \quad \text{(Eq. 13)}$$

We get a virtual object of height  $h'_1$  at  $d'_1$

Now consider the rays travelling inside the lens from the virtual object. Apply spherical surface law now to the second surface. This time  $n' = 1$ ,  $n = n_l$ ,  $l \rightarrow d'_2$ ,  $l' \rightarrow d_2$

$$\boxed{\frac{1}{d_2} - \frac{n_l}{d'_2} = \frac{1 - n_l}{R_2}} \quad \text{(Eq. 14)}$$

The thin lens approximation is that the lens thickness is negligible, so that  $d'_2 \cong d'_1$ . Using this in Eq 13, then substituting in Eq. 14,

$$\boxed{\frac{1}{d_2} - \frac{1}{d_1} = \frac{1 - n_l}{R_2} + \frac{n_l - 1}{R_1}}$$

This is the Gaussian lens law, with the focal length identified as:

$$\frac{1}{f} = (n_l - 1) \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \quad (\text{Eq. 15})$$

This is called the lensmaker's equation.

We conclude that a lens with 2 spherical surfaces satisfies the Gaussian lens law, but only under 2 important approximations

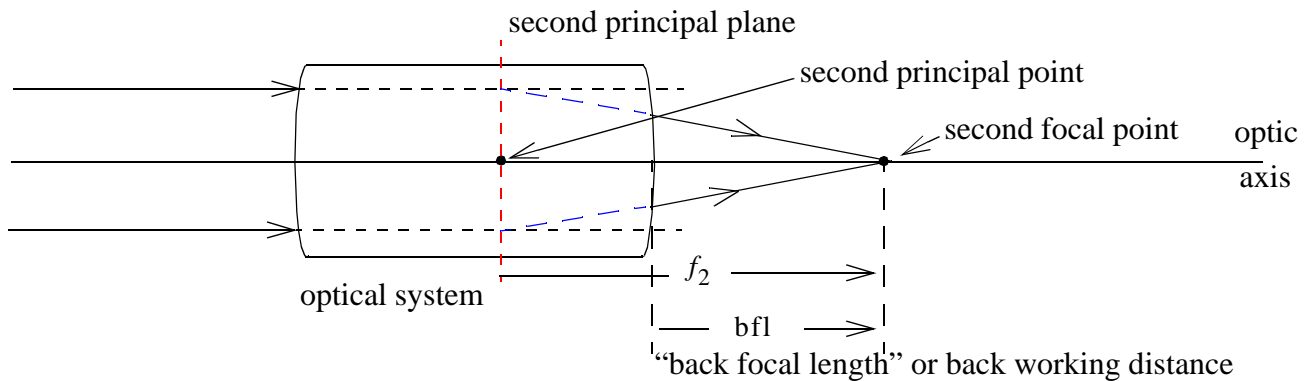
- Paraxial approximation
- Thin-lens approximation

**Thick lens or compound lens systems**

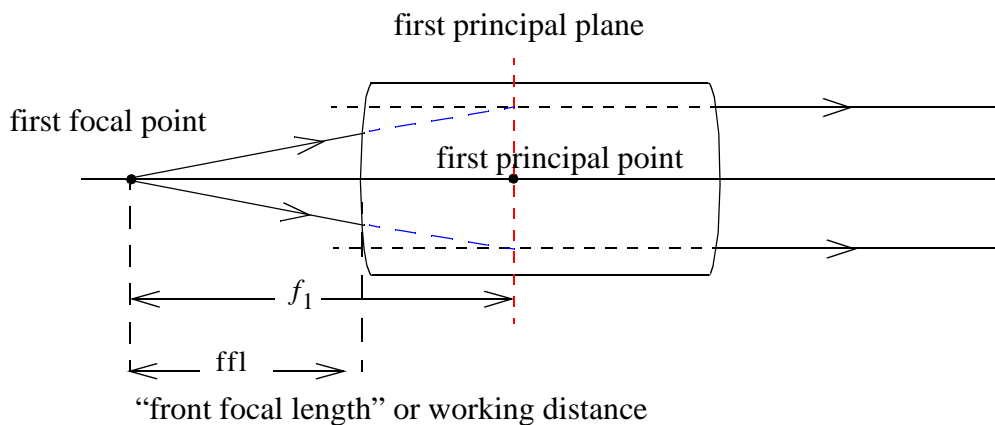
[Reading assignment: Jenkins and White, Sections 5.1-5.6]

Any symmetric optical system consisting of lenses and spaces can be generalized.

Light rays entering from the left, parallel to the optic axis, come to a focus, at the “second focal point”



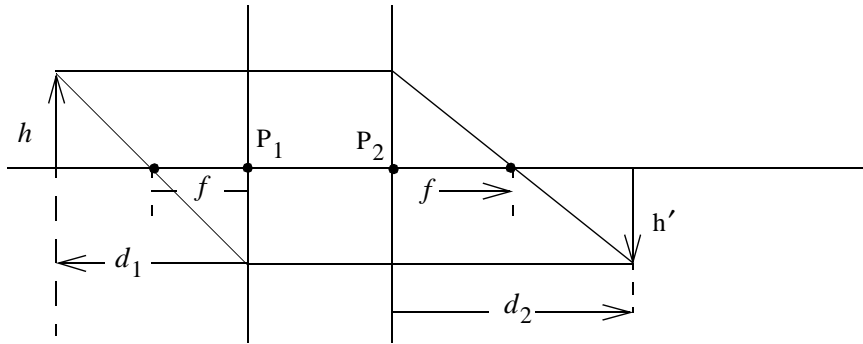
Now, we take the rays entering the system and those emerging from the system and extend them. They intersect on a plane called the “Second principal plane”. Similarly, the first focus and “first principal plane” are defined for rays emerging from the system parallel to the axis, which all emanate from a point.



## LENSES AND IMAGING

We define  $f_2$  as the distance from the second principal plane to the second focal point. Similarly we define  $f_1$  as the distance from the first principal plane to the first focal point. For a system immersed in air (same index on both sides),  $f_1 = f_2$ .

With these definitions, the Gaussian lens law applies as follows:

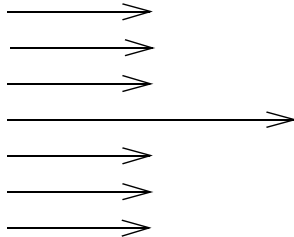


With this geometry, all other relations now apply:

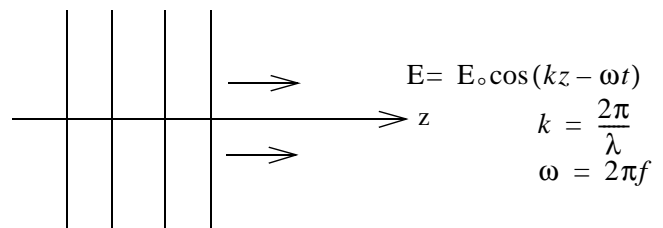
$$\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1} ; m = \frac{d_2}{d_1} ; \bar{m} = m^2$$

### Wave optics of lenses

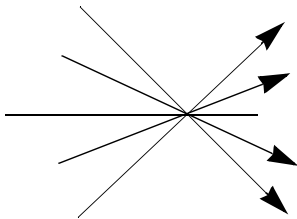
Set of rays parallel to axis



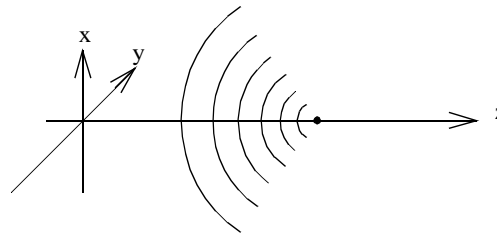
Plane Wave



Rays converging to a focus

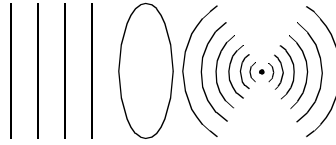


converging spherical wave



## LENSES AND IMAGING

At a given  $z$ -plane, the spherical wave has constant phase around circles. The form of the spherical wave is  $\cos\left[-\frac{k(x^2 + y^2)}{2z_0}\right]$  for a spherical wave converging to the point  $z_0$  on the axis. A lens modifies the wave front, for example from planar to spherical.

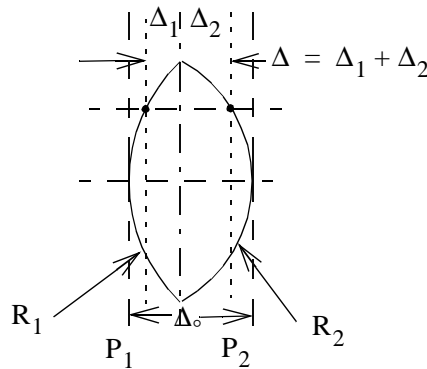


How does this happen?

### Optical path length:

Optical waves travel more slowly in the glass since  $n > 1$ . In glass, the wave is delayed by an amount as if it travelled a distance  $nl$  in free space. If  $l = l(x,y)$  [or  $n = n(x,y)$ ] then the delay varies with  $(x,y)$  so the wavefront gets distorted.

We can analyze the lens in terms of its phase-delay. The light propagates in the glass as  $\cos(knz) = \cos\phi$ , where  $\phi = knz$  is the phase delay.



In propagating from plane  $P_1$  to  $P_2$ , the light travels a distance  $\Delta = \Delta_1 + \Delta_2$  in the glass and a distance  $\Delta_0 - \Delta$  in air, where  $\Delta_0$  is the thickness at the thickest part of the lens. The phase delay depends on  $(x, y)$ :

$$\phi(x, y) = kn\Delta(x,y) + k[\Delta_0 - \Delta(x, y)]$$

$$= k\Delta_0 + k(n - 1)\Delta(x,y)$$



$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$f$  is the focal length of the lens. This expression is identical to what we found from the ray optics analysis.

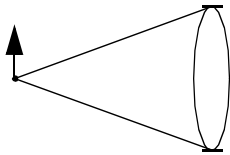
## Stop and Apertures

[Reading assignment: Jenkins and White, Chapter 7]

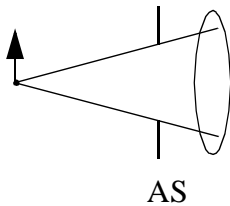
### Aperture Stop

Every optical system has same component that limits the light cone that is accepted from an axial object.

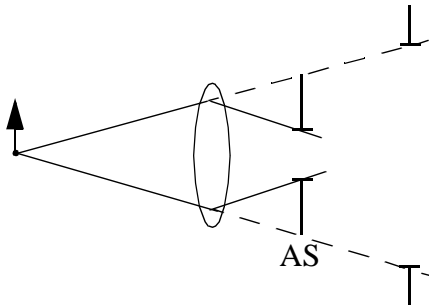
Simple case - single lens



could be the diameter “clear aperture” of the lens.



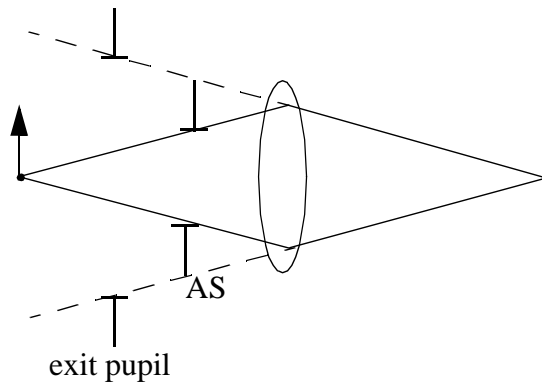
could be a physical aperture placed somewhere in the optical path.



entrance pupil

- Entrance pupil:  
Image of the aperture stop as seen from the object side. Defines the cone of light accepted by the optic.  
The importance of the entrance pupil is that the brightness of the image depends on this cone angle. The larger the acceptance angle, the more light that is collected from each object point, and hence the brighter the image.
- Exit pupil:

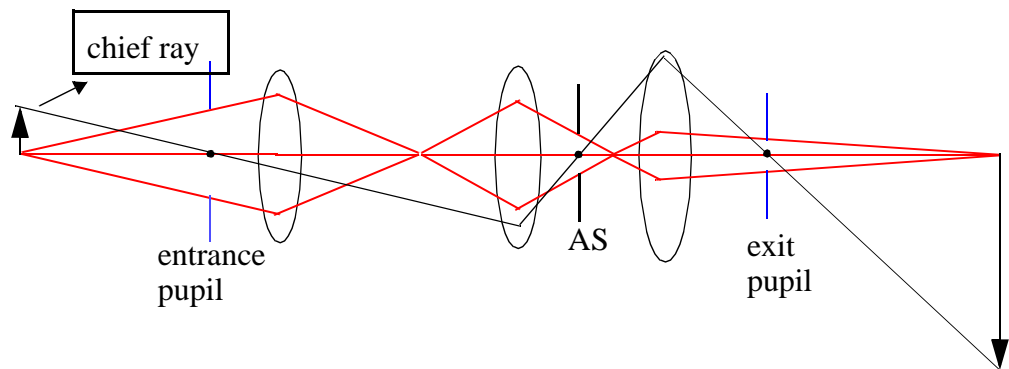
Image of the aperture stop, as seen from the image side of the optic.



The exit pupil defines the cone angle of light converging to the image point. Later, we will see that this is important in determining the image resolution that is set by diffraction.

### Entrance and Exit Pupils are Images of each other

The entrance pupil is the image of the stop. The exit pupil is also an image of the stop. So the entrance and exit pupils must also be images of each other. The pupils define the amount of light accepted by and emitted from the optical system.



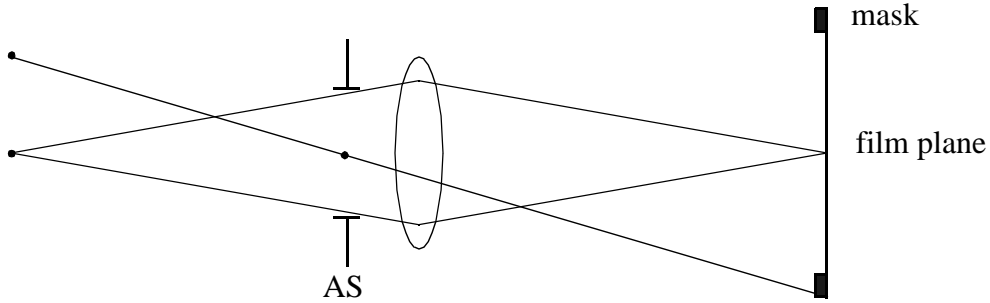
### Chief Ray or Principle Ray

From a given object point is the ray that passes through the center of the pupils.

### Field Stop

Another stop in the system limits the extent of the object/image sizes. The chief ray from an object point is blocked by the field stop.

Simple case: a mask at the object or image plane.



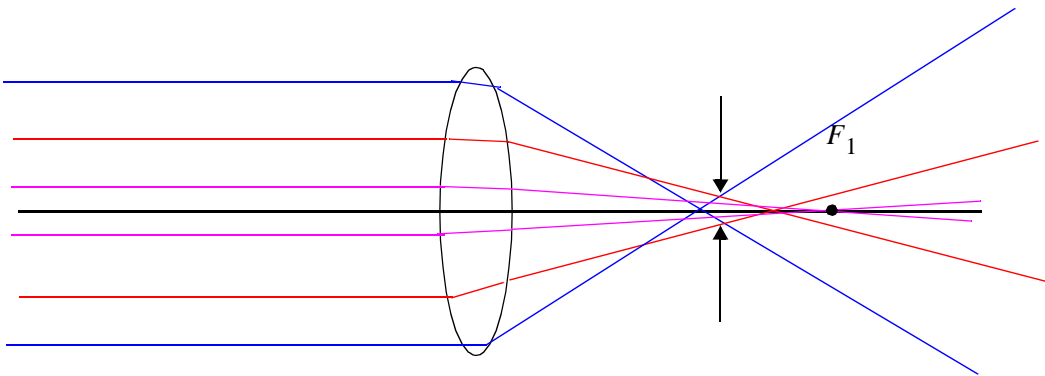
The field stop might also be set by a diaphragm somewhere in the optical path.

- Entrance window: Image of the field stop at the object plane.
- Exit window: Image of the field stop at the image plane.

### Aberrations

[Reading assignment: Jenkins and White, Sections 9.1-9.11]

As we have seen, spherical lenses only obey Gaussian lens law in the paraxial approximation. Deviations from this ideal are called aberrations.



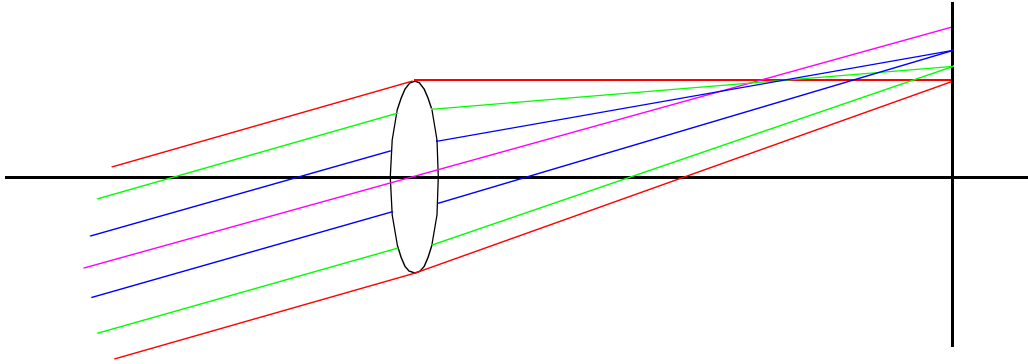
Rays toward the edge of the pupil (even parallel to the axis) violate the paraxial condition on the incidence angle at the first surface. They focus closer (for biconvex lens) than  $F_1$ . No truly sharp focus occurs. The least blurred spot (smallest disc) is called circle of least confusion, or best focus. This form of symmetric aberration is spherical aberration.

There are many forms of aberration.

Coma: Variation of magnification with aperture.

## LENSES AND IMAGING

Rays passing through edge portions of the pupil are imaged at a different height than those passing through the center.



Map of Rays in Pupil

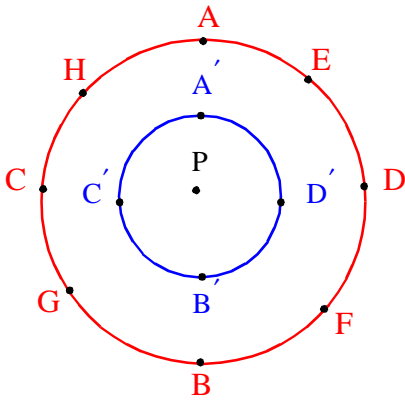
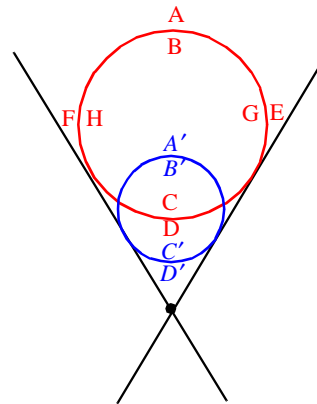
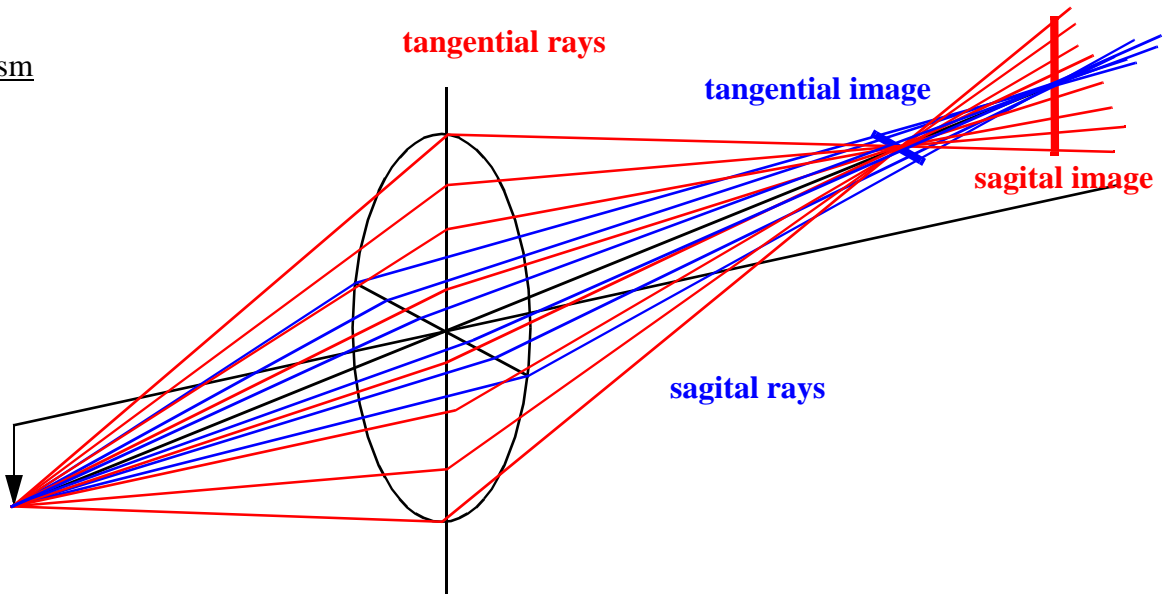


Image Plane

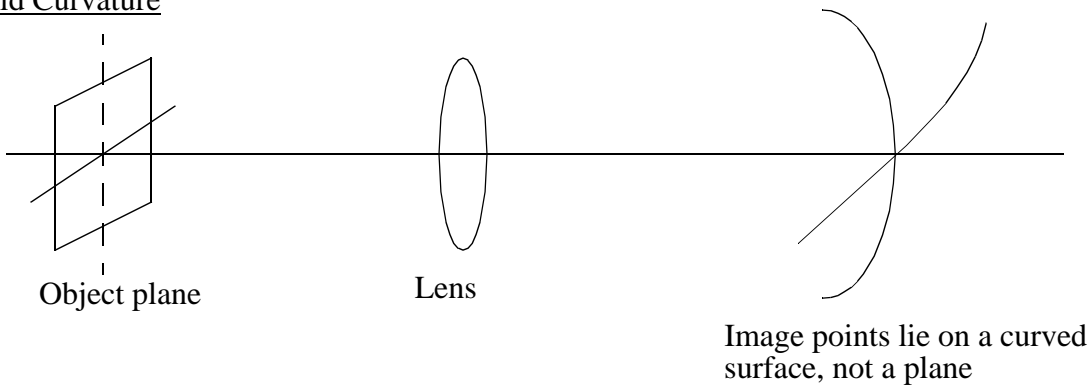


Astigmatism



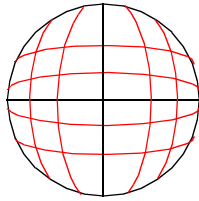
In astigmatism the tangential and sagittal images do not coincide. There are 2 line images with a circle of least confusion in the middle.

Field Curvature

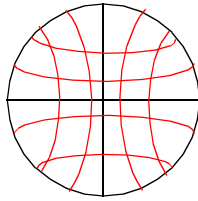


Positive lenses give inward curvature  
negative lenses give backward curvature.

Distortion: Field dependent magnification



Barrel distortion



Pincushion distortion

**Five Primary Aberrations**

Spherical, coma, astigmatism, field curvature, distortion

**Wave Front Aberration**

In a wave-optics picture, the thin lens is represented by phase delay.

$$\phi(x, y) = -k \frac{x^2 + y^2}{2} = -k\Delta(x, y)$$

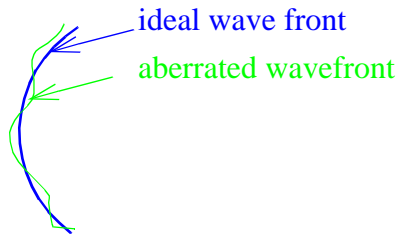
Which gives Gaussian imaging. Aberrations modify  $\phi$ . A spherical lens only gives this  $\phi$  in the paraxial approximation.

- For a complex optical system, we can collect the effects of all the lenses and represent them as a phase delay in the exit pupil. Usually, we subtract the quadratic phase to find the aberration. The residual is called the wave front error; or wfe

$$\Delta(x, y) = -\frac{x^2 + y^2}{2f} + W(x, y)$$

Aberration wfe

$\Delta(x, y)$  usually depends on the field coordinate. In other words, the aberrations can vary depending on where you are in the field of view.



Expressed in this way, the primary aberrations are written as:

Spherical aberration:  $W(x, y) = A_s \rho^4$

## LENSES AND IMAGING

Coma:  $A_c \rho^3 h' \cos \theta$

Astigmatism:  $A_a \rho^2 h'^2 \cos^2 \theta$

Field Curvature:  $A_d \rho^2 h'^2$

Distortion:  $A_t h'^3 \rho \cos \theta$

$\rho$  : is normalized radial coordinate in the pupil

$h'$  : image height

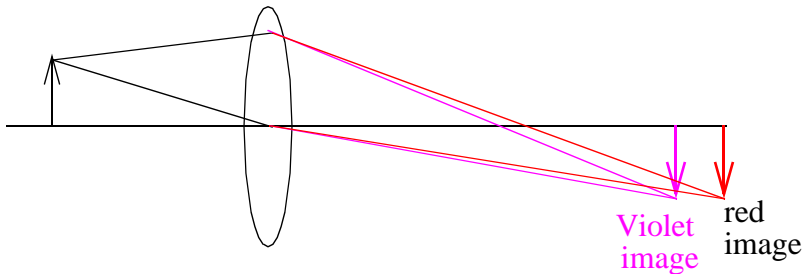
Monochromatic Aberrations: All of the preceding discussion refers to aberrations that do not depend on wavelength.

Chromatic Aberrations: Dependence of wavefront on wavelength.

Consider the simple thin lens equation:

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

The index  $n$  is generally  $\lambda$  dependent,  $n(\lambda)$ , so  $f$  is  $\lambda$  dependent.



Change in image distance: longitudinal chromatic aberration

Change in magnification: lateral color. Lateral color is usually more noticeable

Achromat: lens designed to cancel chromatic aberration.

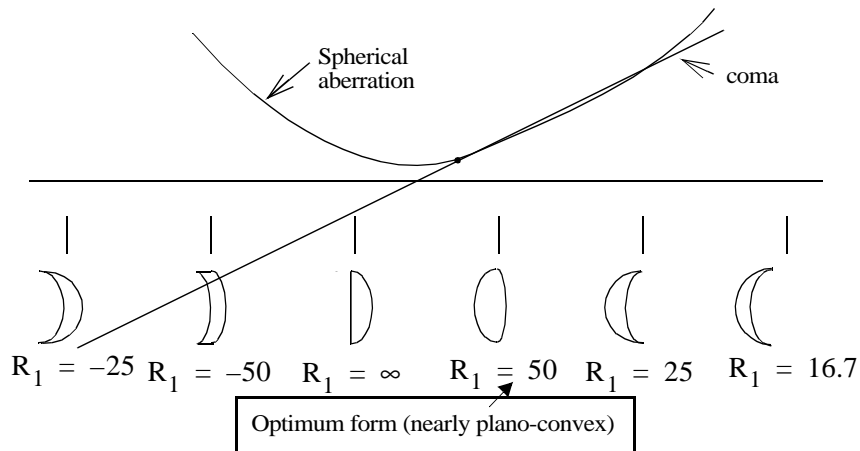
Lens Design:

- The general problem of lens design involves cancelling aberrations
- Aberration depends on the lens index, as well as the surface radii.
- Complex lens systems can minimize aberrations

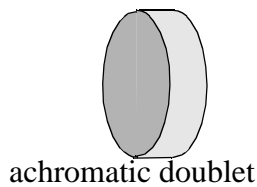
Simple singlet case: For a given desired focal length, there is freedom to choose one of the radii for a singlet. The spherical aberration and coma depend on the particular choice, so these aberrations can be minimized by the design form. This is illustrated in the following diagram:

## LENSES AND IMAGING

design: 100mm focal length  $\pm 17^\circ$  field.

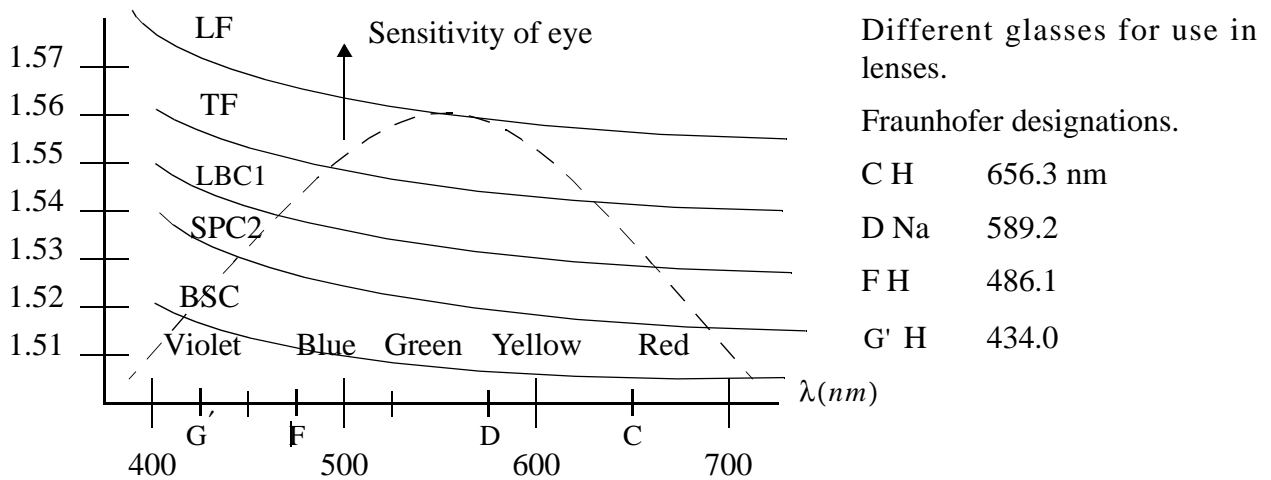


Achromatic doublet. Two elements made from different glass materials

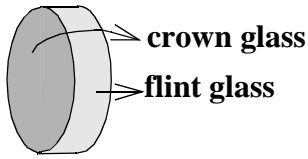


|                   |  |
|-------------------|--|
| positive element: | undercorrected spherical<br>undercorrected chromatic |
| negative element: | both overcorrected                                   |

We generally choose design an achromat to minimize chromatic aberration across the visible part of the spectrum.



**Design of Cemented Doublet Achromat**



The ‘D’ wavelength, near the center of visual brightness curve is chosen as the nominal wavelength for specifying focal length. We then choose 2 indices on either side, for achromatization, for example, ‘C’, and ‘F’.

For 2 thin lenses in contact

$$\frac{1}{f_D} = \frac{1}{f_{D'}} + \frac{1}{f_{D''}}$$

prime: crown glass  
double prime: flint glass

define lens power  $P = \frac{1}{f}$  with  $f$  in meters,

$P$  units are diopeters

$$P_D = P_{D'} + P_{D''}$$

$$= (n_{D'} - 1) \left( \frac{1}{r_1'} - \frac{1}{r_2'} \right) + (n_{D''} - 1) \left( \frac{1}{r_1''} - \frac{1}{r_2''} \right)$$

Define  $K' = \left( \frac{1}{r_1'} - \frac{1}{r_2'} \right)$   $K'' = \left( \frac{1}{r_1''} - \frac{1}{r_2''} \right)$

$$P_D = (n_{D'} - 1)K' + (n_{D''} - 1)K''$$

$$P_F = (n_{F'} - 1)K' + (n_{F''} - 1)K''$$

$$P_C = (n_{C'} - 1)K' + (n_{C''} - 1)K''$$

Achromatic design means we make  $P_C = P_F$

$$(n_{F'} - 1)K' + (n_{F''} - 1)K'' = (n_{C'} - 1)K' + (n_{C''} - 1)K''$$

Simplifies to:

$$\frac{K'}{K''} = \frac{n_{F''} - n_{C''}}{n_{F'} - n_{C'}}$$

For normal dispersion  $K'$  has the opposite sign from  $K''$ . One lens must be positive one lens must be negative.

For the center of the spectrum (D-line)

$$P_{D'} = (n_{D'} - 1)K'$$

$$P_{D''} = (n_{D''} - 1)K''$$

, so

$$\frac{K'}{K''} = -\frac{(n_D'' - 1)P_D'}{(n_D' - 1)P_D''}$$

Combining results, we find:

$$\frac{P_D''}{P_D'} = -\frac{(n_D'' - 1)(n_F'' - n_C'')}{(n_D' - 1)(n_F' - n_C')} \equiv -\frac{v''}{v'} \quad (\text{Eq. 1})$$

$v \equiv \frac{n_D - 1}{n_F - n_C}$  is a property of a given glass called the “dispersion constant”

$V = 1/v$  is called the “dispersive power” or V-number. Glass manufacturers spec these numbers for use by designers. Now, from Eq. 1,

$$\frac{P_D''}{v''} + \frac{P_D'}{v'} = 0 \quad (\text{Eq. 2})$$

and

$$P_D' = P_D \frac{v'}{v' - v''} \quad P_D'' = -P_D \frac{v''}{v' - v''} \quad (\text{Eq. 3})$$

Eqs. 2 and 3 are the design equations.

- Design starts with desired  $f_D, P_D$
- Next choose your glass materials, i.e.  $v', v''$
- Find  $P_D', P_D''$  from Eq. 3, then get  $K', K''$
- Choose radii (still some freedom left in choice of radii for minimization of monochromatic aberrations). A common, simple choice is to make the crown lens biconvex, and to cement the two lenses together, with no gap. This means:

$$r_2' = -r_1' \quad \text{biconvex}$$

$$r_1'' = r_2' = -r_1' \quad \text{cemented}$$

Then  $r_2''$  is set by the constraint of Eq 3.

For crown glass facing parallel light, this gives a good design to minimize spherical and coma. It can be fine tuned by careful choice of  $v', v''$

Example: Design 10cm focal length cemented doublet using the crown and flint glasses.  $P_D = 10D$

|       | $n_C$   | $n_D$  | $n_F$   | $n_G'$  |
|-------|---------|--------|---------|---------|
| crown | 1.50868 | 1.511  | 1.51673 | 1.52121 |
| flint | 1.61611 | 1.6210 | 1.63327 | 1.64369 |

## LENSES AND IMAGING

$$v' = 63.4783 \quad v'' = 36.1888$$

$$P_D' = 23.2611D \quad P_D'' = -13.2611D$$

note  $\boxed{P_D = P_D' + P_D'' = 10D}$  (checks!)

Using biconvex for positive element  $r_1' = -r_2'$

$$K' = \frac{2}{r_1'} = \frac{P_D'}{n_D - 1} = 45.5207$$

$$r_1' = 0.043961\text{m} = 4.3961\text{cm}$$

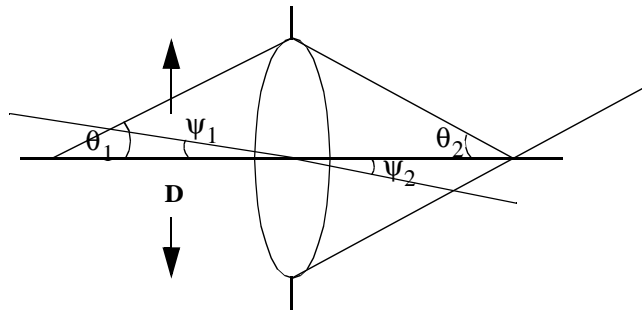
with  $r_1' = -r_1''$ ,  $K'' = -\frac{1}{r_1} - \frac{1}{r_2''} = \frac{P_D''}{n_D'' - 1} = -21.3544$

So,  $\boxed{r_2'' = -71.13\text{cm}}$

This completes the design

### Resolution limit of an optical system

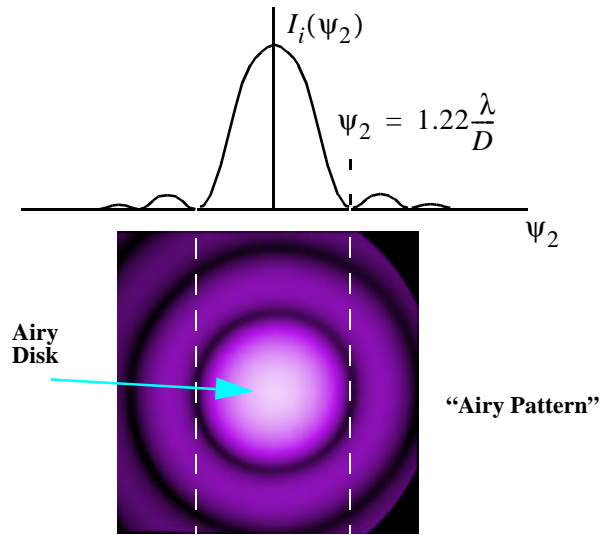
[Reading assignment: Jenkins and White, Sections 15.8-15.10]



Due to diffraction at the aperture stop, the image of a point is slightly blurred. Diffraction theory tells us that the image depends on the shape of the aperture. For a circular aperture:

$$I_i(\psi_2) = I_o \left[ \frac{2J_1\left(\pi \frac{D}{\lambda} \sin \psi_2\right)}{\pi \frac{D}{\lambda} \sin \psi_2} \right]^2$$

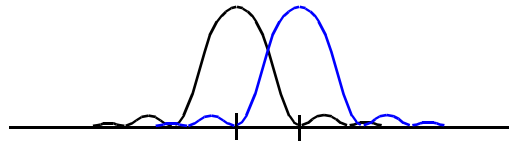
$J_1(x)$  is a special function called the “Bessel Function of the First Kind”.



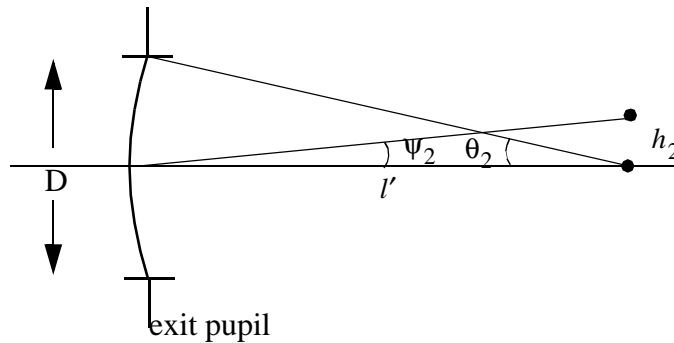
The first zero in the pattern occurs at  $\psi_2 = 1.22 \frac{\lambda}{D}$ .

If 2 points lie close together in the object plane, the Airy patterns will overlap. The criterion for whether the 2 points can be resolved depends on the type of imaging application and it is somewhat arbitrary. A very common criterion is Rayleigh's criterion.

According to Rayleigh's criterion, 2 spots are resolved if the maximum of the pattern from one point falls on the first minimum of the other.



We say that the angular resolution in the image plane is  $1.22\lambda/D$



with  $l'$  the distance to the exit pupil (radius of exit sphere), we have

$$l' = \frac{-D}{2 \sin \theta_2}$$

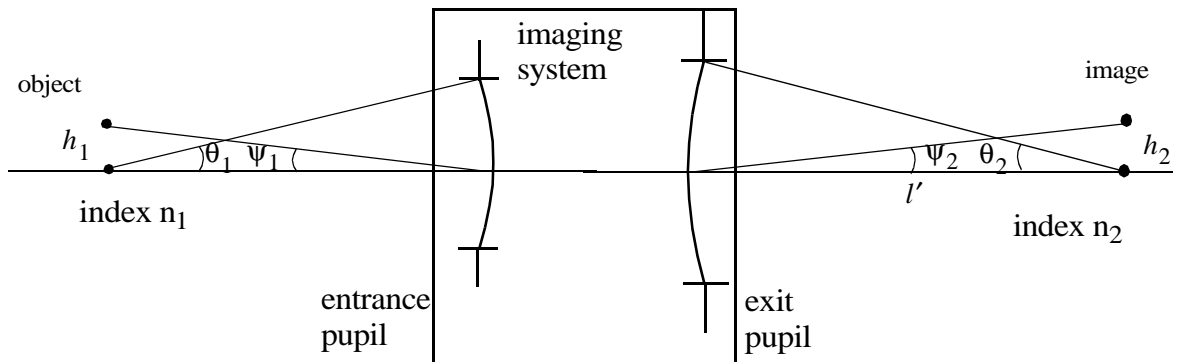
for small  $\psi_2$  (small  $h_2$ ),  $h_2 \cong l' \psi_2$

$$\text{so } h_2 = \frac{-D \psi_2}{2 \sin \theta_2} = \frac{0.61 \lambda}{\sin \theta_2} = \frac{0.61 \lambda}{NA_2}$$

where  $NA_2 \equiv \sin \theta_2$ .

The “Numerical Aperture” or NA is a very important property of an imaging system. It is simply the sine of the half angle subtended by the pupil. Here,  $NA_2$  is the numerical aperture of the exit pupil.

Somewhat more generally, consider a complete imaging system. The entrance pupil subtends an angle  $\theta_1$  with an object of height  $h_1$ ,



A general property of imaging systems holds that:

$$h_1 n_1 \sin \theta_1 = h_2 n_2 \sin \theta_2 .$$

The numerical aperture is generalized if the object and image spaces are immersed in different index of refraction.

$$n \sin \theta_1 \equiv NA_1 . \quad n_2 \sin \theta_2 \equiv NA_2$$

Here  $NA_1$  is the “entrance” numerical aperture and  $NA_2$  is the “exit numerical aperture.”

Once again we have:  $h_2 = \frac{0.61 \lambda}{NA_2}$

For large  $NA \sim 0.6$   $h_2 = \lambda$ , so the resolution  $\sim$  wavelength of light.

Also notice that:

$$\frac{h_2}{h_1} = m = \frac{NA_1}{NA_2}$$

which says that the entrance and exit numerical apertures have a ratio equal to the transverse magnification.