Design of Engineering Experiments
Chapter 7 – Blocking & Confounding in the 2^k

- Text reference, Chapter 7 page 288
- **Blocking** is a technique for dealing with controllable **nuisance** variables
- Two cases are considered
  - Replicated designs
  - Un-replicated designs

Blocking a Replicated Design

- This is the same scenario discussed previously (Chapter 5, Section 5-6)
- There are many situations where it is impossible to perform all runs in a 2k experiment under homogeneous conditions: Maybe single batch of raw material is not large enough for all runs or multiple operators have to run different runs due to work schedules.
- If there are *n* replicates of the design, then each group of replicates is run as a block.
- Each **replicate** is run in one of the **blocks** (time periods, batches of raw material, etc.)
- Runs within the block are **randomized**

Blocking a Replicated Design 2^2 design

Consider the example from Section 6-2; $k = 2$ factors, $n = 3$ replicates

This is the “usual” method for calculating a block sum of squares

Each batch of material is only large enough to run 3 runs, so we’ll run replicates as blocks on material.

Table 7-1 Chemical Process Experiment in Three Blocks

	Block 1	Block 2	Block 3
	(1) = 28	(1) = 25	(1) = 27
	a = 36	a = 32	a = 32
	b = 18	b = 19	b = 23
	ab = 31	ab = 30	ab = 29
Block totals:	$B_1 = 113$	$B_2 = 106$	$B_3 = 111$

$$SS_{Blocks} = \sum_{i=1}^{3} \frac{B_i^2}{4} - \frac{y_{...}^2}{12}$$

$$= 6.50$$

ANOVA for the Blocked Design Page 288

Table 7-2 Analysis of Variance for the Chemical Process Experiment in Three Blocks

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Blocks	6.50	2	3.25		
A (concentration)	208.33	1	208.33	50.32	0.0004
B (catalyst)	75.00	1	75.00	18.12	0.0053
AB	8.33	1	8.33	2.01	0.2060
Error	24.84	6	4.14		
Total	323.00	11			

For this example the effect of blocks is very small

What do you do if you cannot run a all treatment combinations in one Block?

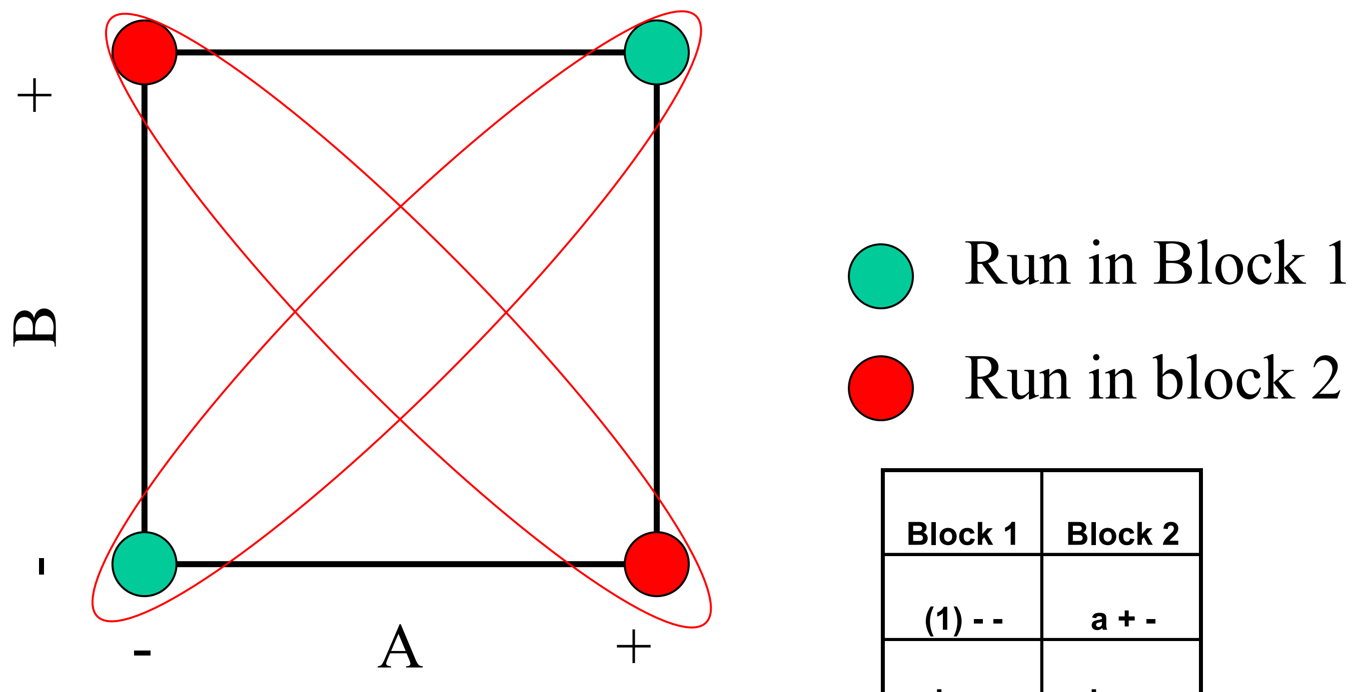
- ***Confounding or (Aliasing)*** is a technique for situations where you cannot perform a complete replicate in one block.
- ***Block size is smaller than total # of treatment combinations in one replicate.***
- This causes the ***higher order interactions*** to be indistinguishable from blocks or ***confounded (Aliased) with blocks***.
- These are also called ***incomplete blocks***.
- Run a 2^k experiment in 2^p blocks ($p < k$)

Confounding in Blocks Simple 2^2 design

- Simple example of a single replicate 2^2 design:
- One batch of material is only large enough to run 2 runs from the design.
- So we have to run the experiment in 2 blocks.
- How do I know which treatment combinations to run in which block?
- We desire to ***confound higher order interactions within a block, which in this case that would be the AB interaction.***
- ***So all same sign A*B products go in same block***

Confounding in BlocksSimple 2^2 design

- Simple example of a single replicate 2^2 design: interaction AB is confounded with the blocks. Confound the highest order interactions with the block



Block 1	Block 2
(1) - -	a + -
ab + +	b - +

Confounding in Blocks Simple 2^2 design

- **2^2 design Blocked: highest order interaction AB sign split between blocks. So AB interaction is confounded with blocks**

run	Treatment name	A level	B level	AB = AxB	Block
1	(1) avg	-	-	+	I
2	a	+	-	-	II
3	b	-	+	-	II
4	ab	+	+	+	I

- **We can use this method to confound the higher order interactions in a 2^k design in two blocks.**
- **See page 290 table 7-4**

2^2 design Blocked: Show $SS_{block} = SS_{AB}$

7-12 Consider the 2^2 design in two blocks with AB confounded. Prove algebraically that $SS_{AB} = SS_{Blocks}$.

If AB is confounded, the two blocks are:

Block 1	Block 2
(1)	a
ab	b
$(1) + ab$	$a + b$

$$SS_{Blocks} = \frac{[(1)+ab]^2 + [a+b]^2}{2} - \frac{[(1)+ab+a+b]^2}{4}$$

$$SS_{Blocks} = \frac{(1)^2 + ab^2 + 2(1)ab + a^2 + b^2 + 2ab}{2}$$

$$-\frac{(1)^2 + ab^2 + a^2 + b^2 + 2(1)ab + 2(1)a + 2(1)b + 2a(ab) + 2b(ab) + 2ab}{4}$$

$$SS_{Blocks} = \frac{(1)^2 + ab^2 + a^2 + b^2 + 2(1)ab + 2ab - 2(1)a - 2(1)b - 2a(ab) - 2b(ab)}{4}$$

$$SS_{Blocks} = \frac{1}{4}[(1)+ab-a-b]^2 = SS_{AB}$$

Other Methods for constructing Blocks page 290 -293

- **Linear Combination Method:**
- $L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots\ldots. + \alpha_k x_k$
- **The value of L will determine which block the treatments go in.**
- x_i **is the ith factor treatment combination**
- α_i **is ith factor's exponent in the effect to be confounded**
- *The equation above for L is called the defining contrast.*
- **For** 2^k **we have** $\alpha_i = 0$ **or +1 and**
- $x_i = 0$ **(low level) or +1 (high level)**

Other Methods for constructing Blocks page 290 -293

- **Linear Combination Method:**
- **$L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots\ldots. + \alpha_k x_k$**
- **Treatment combinations that produce the same value of L value (defining contrast) are placed in the same block.**
- ***Only possible values of L in a 2^k design to be broken into 2 blocks are 0 and +1.***
- **Uses Modulus 2 math to reduce the value of L to 0 or 1 by 2's if L >1.**

- ***For 2^k we have α_1 = 0 or +1 and x_1 = 0 or +1***
- ***2^3 design example: page 291***

- ***x_1 = A ; x_2 = B ; x_3 = C ; using the levels of the factors as: 0 = low level and 1 = the high level***
- ***$\alpha_1 = 1$; $\alpha_2 = 1$; $\alpha_3 = 1$;***
- **Continued……**

Other Methods for constructing Blocks page 290 -293 What is Modulus 2 math?

- **Modulus 2**
- **Excel:**
- **MOD(n,2)**
- **Sign in math is %**
- **As: n%2**

n	MOD(n,2)
1	1
2	0
3	1
4	0
5	1

MOD

See Also

Returns the remainder after number is divided by divisor. The result has the same sign as divisor.

Syntax

MOD(number,divisor)

Number is the number for which you want to find the remainder.

Divisor is the number by which you want to divide number.

Remarks

- If divisor is 0, MOD returns the #DIV/0! error value.
- The MOD function can be expressed in terms of the INT function:

```
MOD(n, d) = n - d*INT(n/d)
```

Example

The example may be easier to understand if you copy it to a blank worksheet.

How?

	A	B
1	**Formula**	**Description (Result)**
2	=MOD(3, 2)	Remainder of 3/2 (1)
3	=MOD(-3, 2)	Remainder of -3/2. The sign is the same as divisor (1)
4	=MOD(3, -2)	Remainder of 3/-2. The sign is the same as divisor (-1)
5	=MOD(-3, -2)	Remainder of -3/-2. The sign is the same as divisor (-1)

Other Methods for constructing Blocks in 2^k designs page 290 -293

- **Linear Combination Method for 2^k design:**
- $L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots\ldots + \alpha_k x_k$
- 2^3 design example: page 291
- x_1 = A level (0 or 1) ; x_2 = B level (0 or 1) ; x_3 = C level (0 or 1)
- $\alpha_1 = 1$; $\alpha_2 = 1$; $\alpha_3 = 1$;
- Defining contrast is: $L = x_1 + x_2 + x_3$
- By these definitions for a 2^3 design this confounds ABC (FACTORS 1,2,3) with the block!

Linear Combination Method for constructing blocks: 2^3 example page 290 -293

- 2^3 example
- $L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots\ldots\ldots + \alpha_k x_k$
- $\alpha_1 = 1$; $\alpha_2 = 1$; $\alpha_3 = 1$;
- Defining contrast is: $L = x_1 + x_2 + x_3$
- (1) : L = 1(0) + 1(0) + 1(0) = 0 = 0 (mod 2)
- (a) : L = 1(1) + 1(0) + 1(0) = 1 = 1 (mod 2)
- (b) : L = 1(0) + 1(1) + 1(0) = 1 = 1 (mod 2)
- (ab) : L = 1(1) + 1(1) + 1(0) = 2 = 0 (mod 2)
- (c) : L = 1(0) + 1(0) + 1(1) = 1 = 1 (mod 2)
- (ac) : L = 1(1) + 1(0) + 1(1) = 2 = 0 (mod 2)
- (bc) : L = 1(0) + 1(1) + 1(1) = 2 = 0 (mod 2)
- (abc) : L = 1(1) + 1(1) + 1(1) = 3 = 1 (mod 2)

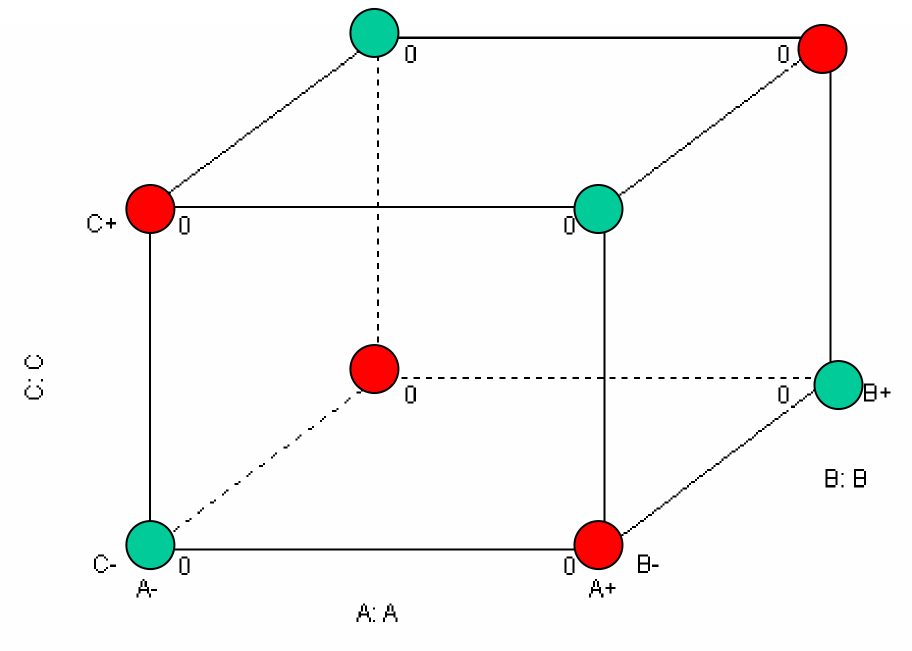
Other Methods for constructing Blocks page 290 -293

- **Linear Combination Method:**
- $L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots\ldots + \alpha_k x_k$
- ***2^3 design example: page 291***
- **Defining contrast is: $L = x_1 + x_2 + x_3$**

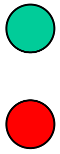
Run #	Treatment Name	A Level	B Level	C Level	L	MOD(L,2)	BLOCK
1	(1) Average	0	0	0	0	0	I
2	a	1	0	0	1	1	II
3	b	0	1	0	1	1	II
4	ab	1	1	0	2	0	I
5	c	0	0	1	1	1	II
6	ac	1	0	1	2	0	I
7	bc	0	1	1	2	0	I
8	abc	1	1	1	3	1	II

Other Methods for constructing Blocks page 290 -293

- *2^3 design example: page 291*
- Defining contrast is: $L = x_1 + x_2 + x_3$



2^3 in 2 blocks	
Block 1	**Block 2**
(1)	*abc*
ac	*a*
ab	*b*
bc	*c*



Run in block 1

Run in block 2

Confounding in Blocks 2^4 Design

- Now consider the **unreplicated** case
- Clearly the previous discussion does not apply, since there is only one replicate
- To illustrate, consider the situation of Example 6-2, Page 248
- This is a 2^4, $n = 1$ replicate

EXAMPLE: Blocking and Confounding:
Example 7-2 page 293 from 6-2 data Un-Replicated

"Recipe Matrix": Tells us how to run the experiment.

Table 6-10 Pilot Plant Filtration Rate Experiment

Run Number	Factor A	Factor B	Factor C	Factor D	Run Label	Filtration Rate (gal/h)
1	−	−	−	−	(1)	45
2	+	−	−	−	*a*	71
3	−	+	−	−	*b*	48
4	+	+	−	−	*ab*	65
5	−	−	+	−	*c*	68
6	+	−	+	−	*ac*	60
7	−	+	+	−	*bc*	80
8	+	+	+	−	*abc*	65
9	−	−	−	+	*d*	43
10	+	−	−	+	*ad*	100
11	−	+	−	+	*bd*	45
12	+	+	−	+	*abd*	104
13	−	−	+	+	*cd*	75
14	+	−	+	+	*acd*	86
15	−	+	+	+	*bcd*	70
16	+	+	+	+	*abcd*	96

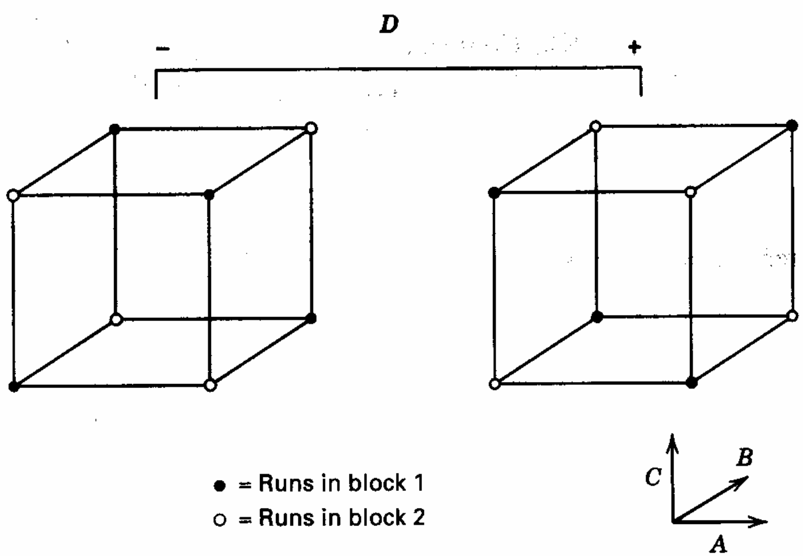
Suppose only 8 runs can be made from one batch of raw material

EXAMPLE: Blocking and Confounding:
The Table of + & - Signs, Example 7-2 page 293

"Calculation Matrix": Contrasts used to calculate "effects".

Table 6-11 Contrast Constants for the 2^4 Design

	A	*B*	*AB*	*C*	*AC*	*BC*	*ABC*	*D*	*AD*	*BD*	*ABD*	*CD*	*ACD*	*BCD*
(1)	−	−	+	−	+	+	−	−	+	+	−	+	−	−
a	+	−	−	−	−	+	+	−	−	+	+	+	+	−
b	−	+	−	−	+	−	+	−	+	−	+	+	−	+
ab	+	+	+	−	−	−	−	−	−	−	−	+	+	+
c	−	−	+	+	−	−	+	−	+	+	−	−	+	+
ac	+	−	−	+	+	−	−	−	−	+	+	−	−	+
bc	−	+	−	+	−	+	−	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	−	−	−	−	−	−	−
d	−	−	+	−	+	+	−	+	−	−	+	−	+	+
ad	+	−	−	−	−	+	+	+	+	−	−	−	−	+
bd	−	+	−	−	+	−	+	+	−	+	−	−	+	−
abd	+	+	+	−	−	−	−	+	+	+	+	−	−	−
cd	−	−	+	+	−	−	+	+	−	−	+	+	−	−
acd	+	−	−	+	+	−	−	+	+	−	−	+	+	−
bcd	−	+	−	+	−	+	−	+	−	+	−	+	−	+
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+



(*a*) Geometric view

Block 1	Block 2
(1) = 25	*a* = 71
ab = 45	*b* = 48
ac = 40	*c* = 68
bc = 60	*d* = 43
ad = 80	*abc* = 65
bd = 25	*bcd* = 70
cd = 55	*acd* = 86
abcd = 76	*abd* = 104

(*b*) Assignment of the 16 runs to two blocks

Figure 7-4 The 2^4 design in two blocks for Example 7-2.

ABCD is Confounded with Blocks (Page 294)

To demonstrate the block effect and the impact on the results the observations in ***block 1 are reduced by 20 units…this is called the simulated "block effect"***

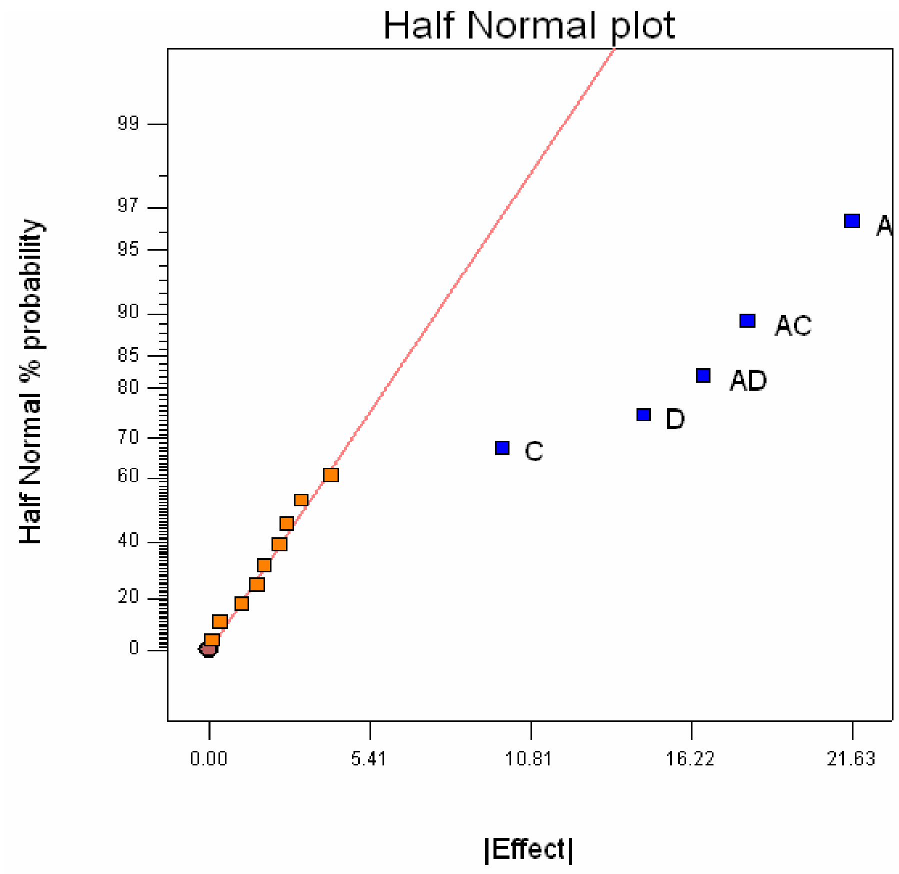
EXAMPLE: Blocking and Confounding: *2^4 Design and block* Table: Pilot Plant Filtration rate Experiment   Example 7-2 page 293

2^4 #	Treatment Name	A Level	B Level	C Level	D Level	L	MOD(L,2)	BLOCK		Filtration rate
1	(1) Average	0	0	0	0	0	0	I	1	25
2	a	1	0	0	0	1	1	II		71
3	b	0	1	0	0	1	1	II		48
4	ab	1	1	0	0	2	0	I	2	25
5	c	0	0	1	0	1	1	II		68
6	ac	1	0	1	0	2	0	I	3	40
7	bc	0	1	1	0	2	0	1	4	60
8	abc	1	1	1	0	3	1	II		65
9	d	0	0	0	1	0	0	I	5	43
10	ad	1	0	0	1	1	1	II		80
11	bd	0	1	0	1	1	1	II		25
12	abd	1	1	0	1	2	0	I	6	104
13	cd	0	0	1	1	1	1	II		55
14	acd	1	0	1	1	2	0	I	7	86
15	bcd	0	1	1	1	2	0	I	8	70
16	abcd	1	1	1	1	3	1	II		76

EXAMPLE: Blocking and Confounding: 2^4 Design and 2 block Example 7-2 page 293-296

Std	Run	Block	Factor 1 A:Temperature C	Factor 2 B:Pressure PSI	Factor 3 C:Formaldehyde %	Factor 4 D:Stirring Rate RPM	Response 1 Filtration Rate gal/hour
1	12	Block 2	-1.00	-1.00	-1.00	-1.00	25
2	1	Block 1	1.00	-1.00	-1.00	-1.00	71
3	6	Block 1	-1.00	1.00	-1.00	-1.00	48
4	14	Block 2	1.00	1.00	-1.00	-1.00	45
5	7	Block 1	-1.00	-1.00	1.00	-1.00	68
6	16	Block 2	1.00	-1.00	1.00	-1.00	40
7	13	Block 2	-1.00	1.00	1.00	-1.00	60
8	5	Block 1	1.00	1.00	1.00	-1.00	65
9	8	Block 1	-1.00	-1.00	-1.00	1.00	43
10	10	Block 2	1.00	-1.00	-1.00	1.00	80
11	11	Block 2	-1.00	1.00	-1.00	1.00	25
12	3	Block 1	1.00	1.00	-1.00	1.00	104
13	15	Block 2	-1.00	-1.00	1.00	1.00	55
14	4	Block 1	1.00	-1.00	1.00	1.00	86
15	2	Block 1	-1.00	1.00	1.00	1.00	70
16	9	Block 2	1.00	1.00	1.00	1.00	76

2^4 Design and 2 block Example 7-2 page 293-296



2^4 Design and 2 block Example 7-2 page page 295 :

Obviously block is significant! Remember we purposely reduced all values in Block 1 by 20 from the original data.

Block Effect = Block + ABCD

Blocking WORKED!

Note: The results obtained in the ANOVA table are the same as original data (not reduced by 20) see page 250 table 6-13

Table 7-7 Analysis of Variance for Example 7-2

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
Blocks (*ABCD*)	1387.5625	1			
A	1870.5625	1	1870.5625	89.76	<0.0001
C	390.0625	1	390.0625	18.72	0.0019
D	855.5625	1	855.5625	41.05	0.0001
AC	1314.0625	1	1314.0625	63.05	<0.0001
AD	1105.5625	1	1105.5625	53.05	<0.0001
Error	187.5625	9	20.8403		
Total	7111.4375	15			

The *ABCD* interaction (or the block effect) is not considered as part of the error term

EXAMPLE: 2 block Effect Estimates page 295

Table 7-6 Effect Estimates for the Blocked 2^4 Design in Example 7-2

Model Term	Regression Coefficient	Effect Estimate	Sum of Squares	Percent Contribution
A	10.81	21.625	1870.5625	26.30
B	1.56	3.125	39.0625	0.55
C	4.94	9.875	390.0625	5.49
D	7.31	14.625	855.5625	12.03
AB	0.062	0.125	0.0625	<0.01
AC	−9.06	−18.125	1314.0625	18.48
AD	8.31	16.625	1105.5625	15.55
BC	1.19	2.375	22.5625	0.32
BD	−0.19	−0.375	0.5625	<0.01
CD	−0.56	−1.125	5.0625	0.07
ABC	0.94	1.875	14.0625	0.20
ABD	2.06	4.125	68.0625	0.96
ACD	−0.81	−1.625	10.5625	0.15
BCD	−1.31	−2.625	27.5625	0.39
Blocks (*ABCD*)		−18.625	1387.5625	19.51

2^4 Design and 2 block Example 7-2 page page 295 :

IF we did not Block>> NOTE ABCD Interaction same as Block!

Response: **Filtration Rate**

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source		Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model		6923.38	6	1153.90	55.37	< 0.0001	significant
	A	*1870.56*	*1*	*1870.56*	*89.76*	*< 0.0001*	
	C	*390.06*	*1*	*390.06*	*18.72*	*0.0019*	
	D	*855.56*	*1*	*855.56*	*41.05*	*0.0001*	
	AC	*1314.06*	*1*	*1314.06*	*63.05*	*< 0.0001*	
	AD	*1105.56*	*1*	*1105.56*	*53.05*	*< 0.0001*	
	ABCD	*1387.56*	*1*	*1387.56*	*66.58*	*< 0.0001*	
Residual		187.56	9	20.84			
Cor Total		7110.94	15				

The Model F-value of 55.37 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Confounding in Blocks 7-5

- More than two blocks (page 296)
  - The two-level factorial can be confounded in 2, 4, 8, … (2^p, $p > 1$) blocks
  - For **four** blocks, select **two** effects to confound, automatically confounding a **third** effect: ***See table page 298***
  - See example, page 296

Confounding in Blocks 7-5 page 296 complicated case 4 blocks

- The two-level factorial can be confounded in 2, 4, 8, … (2^p, $p > 1$) blocks
- For **four** blocks, select **two** effects to confound
- We now look at pairs of defining contrast values L1 and L2 to figure out which block a treatment falls in.

- **Linear Combination Method:**
- $L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots\ldots + \alpha_k x_k$
- **2^5 design example: page 296 (FACTORS: ABCDE or 12345)**

Confounding in Blocks 7-5 page 296 complicated case 4 blocks

- **EXAMPLE: We want to confound interactions ADE and BCE with blocks.**

- **NOTE: We could have selected any interaction to confound with the block!**

- **Defining contrasts for this EXAMPLE are:**
- **$L1 = x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)**
- **$L2 = x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)**

- **With the technique defined here we also confound the generalized interaction as: ADE x BCE = $ABCDE^2$ = ABCD.**

- **So ABCD interaction is also confounded with the blocks.**
- ***See table page 298***

Confounding in Blocks 7-5 page 296 complicated case>> 4 blocks

- **Defining contrasts for this EXAMPLE are:**
- **$L1 = x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)**
- **$L2 = x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)**

- **CONSTRUCTING the BLOCKS**

2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	ADE L1	BCE L2	MOD(L1,2)	MOD(L2,2)	BLOCK
1	(1) Average	0	0	0	0	0	0	0	0	0	Block I
2	a	1	0	0	0	0	1	0	1	0	Block II
3	b	0	1	0	0	0	0	1	0	1	Block III
4	ab	1	1	0	0	0	1	1	1	1	Block IV
5	c	0	0	1	0	0	0	1	0	1	Block III
6	ac	1	0	1	0	0	1	1	1	1	Block IV
7	bc	0	1	1	0	0	0	2	0	0	Block I
8	abc	1	1	1	0	0	1	2	1	0	Block II
9	d	0	0	0	1	0	1	0	1	0	Block II
10	ad	1	0	0	1	0	2	0	0	0	Block I
11	bd	0	1	0	1	0	1	1	1	1	Block IV
12	abd	1	1	0	1	0	2	1	0	1	Block III
13	cd	0	0	1	1	0	1	1	1	1	Block IV
14	acd	1	0	1	1	0	2	1	0	1	Block III
15	bcd	0	1	1	1	0	1	2	1	0	Block II
16	abcd	1	1	1	1	0	2	2	0	0	Block I

Confounding in Blocks 7-5 page 296 complicated case

4 blocks

- **Defining contrasts for this EXAMPLE are:**
- **L1 = $x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)**
- **L2 = $x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)**
- **CONSTRUCTING the BLOCKS**

							ADE	BCE			
2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	L1	L2	MOD(L1,2)	MOD(L2,2)	BLOCK
17	e	0	0	0	0	1	1	1	1	1	Block IV
18	ae	1	0	0	0	1	2	1	0	1	Block III
19	be	0	1	0	0	1	1	2	1	0	Block II
20	abe	1	1	0	0	1	2	2	0	0	Block I
21	ce	0	0	1	0	1	1	2	1	0	Block II
22	ace	1	0	1	0	1	2	2	0	0	Block I
23	bce	0	1	1	0	1	1	3	1	1	Block IV
24	abce	1	1	1	0	1	2	3	0	1	Block III
25	de	0	0	0	1	1	2	1	0	1	Block III
26	ade	1	0	0	1	1	3	1	1	1	Block IV
27	bde	0	1	0	1	1	2	2	0	0	Block I
28	abde	1	1	0	1	1	3	2	1	0	Block II
29	cde	0	0	1	1	1	2	2	0	0	Block I
30	acde	1	0	1	1	1	3	2	1	0	Block II
31	bcde	0	1	1	1	1	2	3	0	1	Block III
32	abcde	1	1	1	1	1	3	3	1	1	Block IV

Confounding in Blocks 7-5 page 296 complicated case

2^5 design broken into 4 blocks

- **Defining contrasts for this EXAMPLE are:**
- **L1 = $x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)**
- **L2 = $x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)**
- **CONSTRUCTING the BLOCKS**

• BLOCK I

2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	L1	L2	MOD(L1,2)	MOD(L2,2)	BLOCK
1	(1) Average	0	0	0	0	0	0	0	0	0	Block I
7	bc	0	1	1	0	0	0	2	0	0	Block I
10	ad	1	0	0	1	0	2	0	0	0	Block I
16	abcd	1	1	1	1	0	2	2	0	0	Block I
20	abe	1	1	0	0	1	2	2	0	0	Block I
22	ace	1	0	1	0	1	2	2	0	0	Block I
27	bde	0	1	0	1	1	2	2	0	0	Block I
29	cde	0	0	1	1	1	2	2	0	0	Block I

Confounding in Blocks 7-5 page 296 complicated case>>

2^5 design broken into 4 blocks

- Defining contrasts for this EXAMPLE are:
- $L1 = x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)
- $L2 = x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)
- CONSTRUCTING the BLOCKS

- # BLOCK II

2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	L1	L2	MOD(L1,2)	MOD(L2,2)	BLOCK
2	a	1	0	0	0	0	1	0	1	0	Block II
8	abc	1	1	1	0	0	1	2	1	0	Block II
9	d	0	0	0	1	0	1	0	1	0	Block II
15	bcd	0	1	1	1	0	1	2	1	0	Block II
19	be	0	1	0	0	1	1	2	1	0	Block II
21	ce	0	0	1	0	1	1	2	1	0	Block II
28	abde	1	1	0	1	1	3	2	1	0	Block II
30	acde	1	0	1	1	1	3	2	1	0	Block II

Confounding in Blocks 7-5 page 296 complicated case>>

2^5 design broken into 4 blocks

- **Defining contrasts for this EXAMPLE are:**
- **L1 = $x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)**
- **L2 = $x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)**
- **CONSTRUCTING the BLOCKS**

- **BLOCK III**

2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	L1	L2	MOD(L1,2)	MOD(L2,2)	BLOCK
3	b	0	1	0	0	0	0	1	0	1	Block III
5	c	0	0	1	0	0	0	1	0	1	Block III
12	abd	1	1	0	1	0	2	1	0	1	Block III
14	acd	1	0	1	1	0	2	1	0	1	Block III
18	ae	1	0	0	0	1	2	1	0	1	Block III
24	abce	1	1	1	0	1	2	3	0	1	Block III
25	de	0	0	0	1	1	2	1	0	1	Block III
31	bcde	0	1	1	1	1	2	3	0	1	Block III

Confounding in Blocks 7-5 page 296 complicated case>>

2^5 design broken into 4 blocks

- Defining contrasts for this EXAMPLE are:
- L1 = $x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)
- L2 = $x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)
- CONSTRUCTING the BLOCKS

• BLOCK IV

2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	L1	L2	MOD(L1,2)	MOD(L2,2)	BLOCK
4	ab	1	1	0	0	0	1	1	1	1	Block IV
6	ac	1	0	1	0	0	1	1	1	1	Block IV
11	bd	0	1	0	1	0	1	1	1	1	Block IV
13	cd	0	0	1	1	0	1	1	1	1	Block IV
17	e	0	0	0	0	1	1	1	1	1	Block IV
23	bce	0	1	1	0	1	1	3	1	1	Block IV
26	ade	1	0	0	1	1	3	1	1	1	Block IV
32	abcde	1	1	1	1	1	3	3	1	1	Block IV

Block design construction: 4 blocks

- Defining contrasts for this EXAMPLE are:
- L1 = $x_1 + x_4 + x_5$ >>> Confounds ADE (1,4,5)
- L2 = $x_2 + x_3 + x_5$ >>> Confounds BCE (2,3,5)

- General procedure for constructing a 2k factorial design in 4 blocks:

- 1. Determine 2 effects to confound to generate the blocks. Typically use three-factor interactions instead of 2 factor which are typically of interest. i.e. You would not want to confound 2 factor interactions as you cannot distinguish their effect.
- Use care when selecting the two effects confound!
- Remember using the two blocking effect automatically also confound their interaction.
- 2. Construct the design using the defining contrasts L1 and L2

Confounding the 2^k Factorial Design in 2^p Blocks 7-6 page 297-299

- The two-level factorial can be confounded in 2, 4, 8, … (2^p, $p > 1$) blocks
- ***k = # factors***
- ***p = # effects to confound and defining contrasts***
- We can use the above technique to construct a 2^k factorial design confounded in 2^p Blocks ($k > p$), where every block contains exactly 2^{k-p} runs
- ***We select p independent effects to be confounded.***
- Independent means that none of the effects chosen are the generalized interaction of the others (i.e. ABC, CDE, and ABDE).

Confounding the 2^k Factorial Design in 2^p Blocks 7-6 page 297-299

Blocks are generated using the p defining contrasts :

- **$L_1, L_2, L_3, \ldots\ldots\ldots\ldots L_p$**
- Exactly $2^p - p - 1$ other contrasts will be confounded with the blocks. These "other" contrasts are the generalized interactions of the p independent effects initially selected.
- Once this is done execution and analysis are straight forward.
- ***See table 7-8 page 298***

Confounding the 2^k Factorial Design in 2^p Blocks Table 7-8 page 298

Choice of confounding schemes non-trivial:

EXAMPLE: Generate 8 blocks for a 2^6 design:

64 runs divided into 8 blocks of 8 runs each.

7-11 Consider the 2^6 design in eight blocks of eight runs each with *ABCD*, *ACE*, and *ABEF* as the independent effects chosen to be confounded with blocks. Generate the design. Find the other effects confound with blocks.

Block 1	Block 2	Block 3	Block 4	Block 5	Block 6	Block 7	Block 8
b	*abc*	*a*	*c*	*ac*	(1)	*bc*	*ab*
acd	*d*	*bcd*	*abd*	*bd*	*abcd*	*ad*	*cd*
ce	*ae*	*abce*	*be*	*abe*	*bce*	*e*	*ace*
abde	*bcde*	*de*	*acde*	*cde*	*ade*	*abcde*	*bde*
abcf	*bf*	*cf*	*af*	*f*	*acf*	*abf*	*bcf*
de	*acdf*	*abdf*	*bcdf*	*abcdf*	*bdf*	*cdf*	*adf*
aef	*cef*	*def*	*abcef*	*bcef*	*abef*	*acef*	*ef*
bcdef	*abdef*	*acdef*	*def*	*adef*	*cdef*	*bdef*	*abcdef*

The factors that are confounded with blocks are *ABCD*, *ABEF*, *ACE*, *BDE*, *CDEF*, *BCF*, and *ADF*.

Confounding the 2^k Factorial Design in 2^p Blocks Table 7-8 page 298

EXAMPLE: Generate 8 blocks for a 2^6 design:
64 runs divided into 8 blocks of 8 runs each.
EXCEL generated design. *BLOCKS 1 and 2*

TABLE 7-8 page 298 Problem 7-11

		1	2	3	4	5	6	ABEF	ABCD	ACE				
2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	F Level	L1	L2	L3	MOD(L1,2)	MOD(L2,2)	MOD(L3,2)	BLOCK
3	**b**	**0**	**1**	**0**	**0**	**0**	**0**	**1**	**1**	**0**	**1**	**1**	**0**	**Block 1**
14	**acd**	**1**	**0**	**1**	**1**	**0**	**0**	**1**	**3**	**2**	**1**	**1**	**0**	**Block 1**
21	**ce**	**0**	**0**	**1**	**0**	**1**	**0**	**1**	**1**	**2**	**1**	**1**	**0**	**Block 1**
28	**abde**	**1**	**1**	**0**	**1**	**1**	**0**	**3**	**3**	**2**	**1**	**1**	**0**	**Block 1**
40	**abcf**	**1**	**1**	**1**	**0**	**0**	**1**	**3**	**3**	**2**	**1**	**1**	**0**	**Block 1**
41	**df**	**0**	**0**	**0**	**1**	**0**	**1**	**1**	**1**	**0**	**1**	**1**	**0**	**Block 1**
50	**aef**	**1**	**0**	**0**	**0**	**1**	**1**	**3**	**1**	**2**	**1**	**1**	**0**	**Block 1**
63	**bcdef**	**0**	**1**	**1**	**1**	**1**	**1**	**3**	**3**	**2**	**1**	**1**	**0**	**Block 1**
8	**abc**	**1**	**1**	**1**	**0**	**0**	**0**	**2**	**3**	**2**	0	1	0	Block 2
9	**d**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**0**	0	1	0	Block 2
18	**ae**	**1**	**0**	**0**	**0**	**1**	**0**	**2**	**1**	**2**	0	1	0	Block 2
31	**bcde**	**0**	**1**	**1**	**1**	**1**	**0**	**2**	**3**	**2**	0	1	0	Block 2
35	**bf**	**0**	**1**	**0**	**0**	**0**	**1**	**2**	**1**	**0**	0	1	0	Block 2
46	**acdf**	**1**	**0**	**1**	**1**	**0**	**1**	**2**	**3**	**2**	0	1	0	Block 2
53	**cef**	**0**	**0**	**1**	**0**	**1**	**1**	**2**	**1**	**2**	0	1	0	Block 2
60	**abdef**	**1**	**1**	**0**	**1**	**1**	**1**	**4**	**3**	**2**	0	1	0	Block 2

Confounding the 2^k Factorial Design in 2^p Blocks Table 7-8 page 298

EXAMPLE: Generate 8 blocks for a 2^6 design:
64 runs divided into 8 blocks of 8 runs each.
EXCEL generated design. ***BLOCKS 3 and 4***

	TABLE 7-8 page 298		Problem 7-11											
		1	2	3	4	5	6	**ABEF**	**ABCD**	**ACE**				
2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	F Level	L1	L2	L3	MOD(L1,2)	MOD(L2,2)	MOD(L3,2)	BLOCK
2	a	1	0	0	0	0	0	1	1	1	1	1	1	Block 3
15	bcd	0	1	1	1	0	0	1	3	1	1	1	1	Block 3
24	abce	1	1	1	0	1	0	3	3	3	1	1	1	Block 3
25	de	0	0	0	1	1	0	1	1	1	1	1	1	Block 3
37	cf	0	0	1	0	0	1	1	1	1	1	1	1	Block 3
44	abdf	1	1	0	1	0	1	3	3	1	1	1	1	Block 3
51	bef	0	1	0	0	1	1	3	1	1	1	1	1	Block 3
62	acdef	1	0	1	1	1	1	3	3	3	1	1	1	Block 3
5	c	0	0	1	0	0	0	0	1	1	0	1	1	Block 4
12	abd	1	1	0	1	0	0	2	3	1	0	1	1	Block 4
19	be	0	1	0	0	1	0	2	1	1	0	1	1	Block 4
30	acde	1	0	1	1	1	0	2	3	3	0	1	1	Block 4
34	af	1	0	0	0	0	1	2	1	1	0	1	1	Block 4
47	bcdf	0	1	1	1	0	1	2	3	1	0	1	1	Block 4
56	abcef	1	1	1	0	1	1	4	3	3	0	1	1	Block 4
57	def	0	0	0	1	1	1	2	1	1	0	1	1	Block 4

Confounding the 2^k Factorial Design in 2^p Blocks Table 7-8 page 298

EXAMPLE: Generate 8 blocks for a 2^6 design:
64 runs divided into 8 blocks of 8 runs each.
EXCEL generated design. *BLOCKS 5 and 6*

	TABLE 7-8 page 298		Problem 7-11											
		1	2	3	4	5	6	ABEF	ABCD	ACE				
2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	F Level	L1	L2	L3	MOD(L1,2)	MOD(L2,2)	MOD(L3,2)	BLOCK
6	ac	1	0	1	0	0	0	1	2	2	1	0	0	Block 5
11	bd	0	1	0	1	0	0	1	2	0	1	0	0	Block 5
20	abe	1	1	0	0	1	0	3	2	2	1	0	0	Block 5
29	cde	0	0	1	1	1	0	1	2	2	1	0	0	Block 5
33	f	0	0	0	0	0	1	1	0	0	1	0	0	Block 5
48	abcdf	1	1	1	1	0	1	3	4	2	1	0	0	Block 5
55	bcef	0	1	1	0	1	1	3	2	2	1	0	0	Block 5
58	adef	1	0	0	1	1	1	3	2	2	1	0	0	Block 5
1	(1) Average	0	0	0	0	0	0	0	0	0	0	0	0	Block 6
16	abcd	1	1	1	1	0	0	2	4	2	0	0	0	Block 6
23	bce	0	1	1	0	1	0	2	2	2	0	0	0	Block 6
26	ade	1	0	0	1	1	0	2	2	2	0	0	0	Block 6
38	acf	1	0	1	0	0	1	2	2	2	0	0	0	Block 6
43	bdf	0	1	0	1	0	1	2	2	0	0	0	0	Block 6
52	abef	1	1	0	0	1	1	4	2	2	0	0	0	Block 6
61	cdef	0	0	1	1	1	1	2	2	2	0	0	0	Block 6

Confounding the 2^k Factorial Design in 2^p Blocks Table 7-8 page 298

EXAMPLE: Generate 8 blocks for a 2^6 design:
64 runs divided into 8 blocks of 8 runs each.
EXCEL generated design. *BLOCKS 7 and 8*

	TABLE 7-8 page 298		Problem 7-11											
		1	2	3	4	5	6	ABEF	ABCD	ACE				
2^4 #	Treatment Name	A Level	B Level	C Level	D Level	E Level	F Level	L1	L2	L3	MOD(L1,2)	MOD(L2,2)	MOD(L3,2)	BLOCK
7	bc	0	1	1	0	0	0	1	2	1	1	0	1	Block 7
10	ad	1	0	0	1	0	0	1	2	1	1	0	1	Block 7
17	e	0	0	0	0	1	0	1	0	1	1	0	1	Block 7
32	abcde	1	1	1	1	1	0	3	4	3	1	0	1	Block 7
36	abf	1	1	0	0	0	1	3	2	1	1	0	1	Block 7
45	cdf	0	0	1	1	0	1	1	2	1	1	0	1	Block 7
54	acef	1	0	1	0	1	1	3	2	3	1	0	1	Block 7
59	bdef	0	1	0	1	1	1	3	2	1	1	0	1	Block 7
4	ab	1	1	0	0	0	0	2	2	1	0	0	1	Block 8
13	cd	0	0	1	1	0	0	0	2	1	0	0	1	Block 8
22	ace	1	0	1	0	1	0	2	2	3	0	0	1	Block 8
27	bde	0	1	0	1	1	0	2	2	1	0	0	1	Block 8
39	bcf	0	1	1	0	0	1	2	2	1	0	0	1	Block 8
42	adf	1	0	0	1	0	1	2	2	1	0	0	1	Block 8
49	ef	0	0	0	0	1	1	2	0	1	0	0	1	Block 8
64	abcdef	1	1	1	1	1	1	4	4	3	0	0	1	Block 8

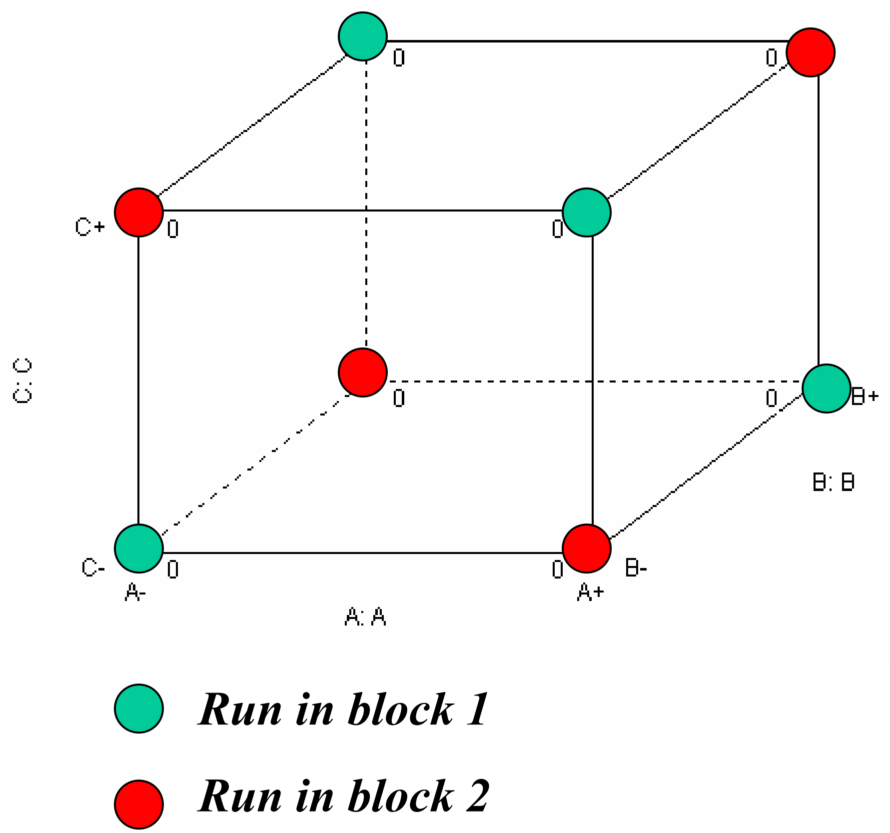
Partial confounding 7-7(page 299)

- ***Unless one has prior knowledge of the error or is willing to assume specific interactions are negligible, one must run replicates to obtain an estimate of error .***
- But one cannot always fully replicate or complete all replicates, so we use blocking.
- If a term like ***ABC in a 2^3 design can be confounded with every block***, then it cannot be distinguished from the other terms. ABC is confounded with each block in the replicate. This type of design is defined to be ***fully or completely confounded***. See Figure 7-3.

Partial confounding 7-7(page 299)

- **Example: ABC in a 2^3 design can be confounded with every block completely confounded.**

2^3 in 2 blocks	
Block 1	**Block 2**
(1)	abc
ac	a
ab	b
bc	c



Partial confounding 7-7(page 299)

- ***Example: ABC in a 2^3 design can be confounded with every block completely confounded.***

Replicate I		Replicate II		Replicate III		Replicate IV	
Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2
(1)	abc	(1)	abc	(1)	abc	(1)	abc
ac	a	ac	a	ac	a	ac	a
ab	b	ab	b	ab	b	ab	b
bc	c	bc	c	bc	c	bc	c

Partial confounding 7-7(page 299)

- Another way to design this experiment would be to ***confound a different interaction with each block replicate.***
- ***Block I : Confound ABC; Block II : Confound AB***
- ***Block III: Confound BC; Block IV: Confound AC***

Replicate I		Replicate II		Replicate III		Replicate IV	
ABC confounded		**AB confounded**		**BC confounded**		**AC confounded**	
Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2
(1)	*abc*	*(1)*	*a*	*(1)*	*b*	*(1)*	*a*
ac	*a*	*c*	*b*	*ab*	*c*	*b*	*c*
ab	*b*	*ab*	*ac*	*bc*	*ab*	*ac*	*ab*
bc	*c*	*abc*	*bc*	*abc*	*ac*	*abc*	*bc*

Partial confounding 7-7(page 299)

- ***Information on ABC can be obtained from Blocks II, III, and IV***

- ***Information on AB from blocks I, III, and IV***
- ***Information on BC from blocks I, II, and IV***
- ***Information on AC from blocks I, II, and III***

Partial confounding 7-7(page 299)

- So we can obtain 75% or 3/4's of the information on he interactions from this design, because they are ***un-confounded*** in only 3 out of four blocks (replicates in this case)
- Yates (1937) defined this ¾ ratio as the relative information for the confounding effects.
- This design technique is called: ***Partial Confounding.***
- ***Example 7-3 on Page 300- 301***
- ***Note calculation of Sum of Squares SS for a confounded interaction only uses the data from the replicate(s) where that interaction is not confounded.***

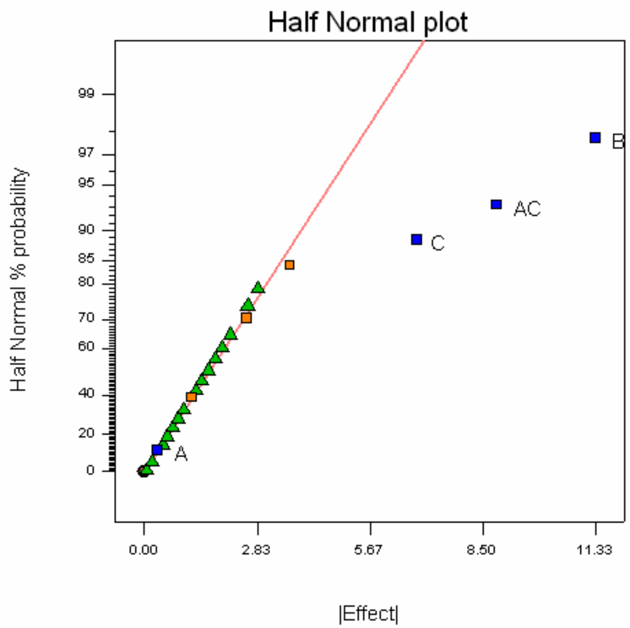
Chapter 7 Examples

- ***Problem: 7-1 Block on production shift use replicates as shifts (problem 6-1):***

Design Summary							
Study Type	Factorial		**Experiments**	24			
Initial Design	2 Level Factorial		**Blocks**	2			
Center Points	0						
Design Model	3FI						
Response	**Name**	**Units**	**Obs**	**Minimum**	**Maximum**	**Trans**	**Model**
Y1	Life	hours	24	22.00	60.00	None	R2FI
Factor	**Name**	**Units**	**Type**	**Low Actual**	**High Actual**	**Low Coded**	**High Coded**
A	Cutting Speed		Numeric	-1.00	1.00	-1.000	1.000
B	Tool Geometry		Numeric	-1.00	1.00	-1.000	1.000
C	Cutting Angle		Numeric	-1.00	1.00	-1.000	1.000

Chapter 7 Examples

- ***Problem: 7-1 Block on production shift . Used replicates as shifts –problem 6-1:***



Response: Life

ANOVA for Selected Factorial Model

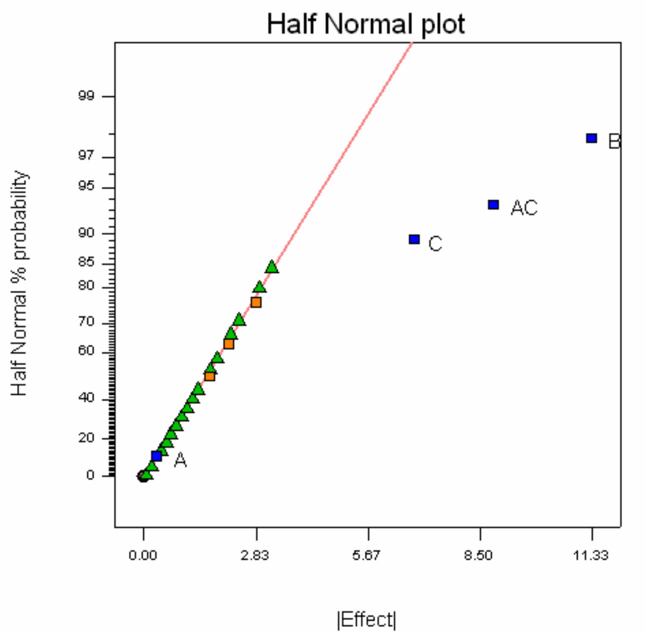
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Block	6.00	1	6.00			
Model	1519.67	4	379.92	12.00	< 0.0001	significant
A	*0.67*	*1*	*0.67*	*0.021*	*0.8862*	
B	*770.67*	*1*	*770.67*	*24.35*	*0.0001*	
C	*280.17*	*1*	*280.17*	*8.85*	*0.0081*	
AC	*468.17*	*1*	*468.17*	*14.79*	*0.0012*	
Residual	569.67	18	31.65			
Lack of Fit	*220.00*	*8*	*27.50*	*0.79*	*0.6259*	*not significant*
Pure Error	*349.67*	*10*	*34.97*			
Cor Total	2095.33	23				

Std. Dev.	5.63	R-Squared	0.7273
Mean	40.83	Adj R-Squared	0.6668
C.V.	13.78	Pred R-Squared	0.5153
PRESS	1012.74	Adeq Precision	9.599

Chapter 7 Examples

- ***Problem: 7-1 Used replicates problem 6-1 Compare:***



Response: Life

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	1519.67	4	379.92	12.54	< 0.0001	significant
A	*0.67*	*1*	*0.67*	*0.022*	*0.8836*	
B	*770.67*	*1*	*770.67*	*25.44*	*< 0.0001*	
C	*280.17*	*1*	*280.17*	*9.25*	*0.0067*	
AC	*468.17*	*1*	*468.17*	*15.45*	*0.0009*	
Residual	575.67	19	30.30			
Lack of Fit	*93.00*	*3*	*31.00*	*1.03*	*0.4067*	*not significant*
Pure Error	*482.67*	*16*	*30.17*			
Cor Total	2095.33	23				

Std. Dev.	5.50	R-Squared	0.7253
Mean	40.83	Adj R-Squared	0.6674
C.V.	13.48	Pred R-Squared	0.5616
PRESS	918.52	Adeq Precision	10.747

Chapter 7 Examples

- ***Problem: 7-9 setup a 2^5 in 4 blocks***

7-9 Consider the data from the 2^5 design in Problem 6-21. Suppose that it was necessary to run this design in four blocks with *ACDE* and *BCD* (and consequently *ABE*) confounded. Analyze the data from this design.

Block 1	Block 2	Block 3	Block 4
(1)	*a*	*b*	*c*
ae	*e*	*abe*	*ace*
cd	*acd*	*bcd*	*d*
abc	*bc*	*ac*	*ab*
acde	*cde*	*abcde*	*ade*
bce	*abce*	*ce*	*be*
abd	*bd*	*ab*	*abcd*
bde	*abde*	*de*	*bcde*

Chapter 7 Examples

- ***Problem: 7-9 setup a 2^5 in 4 blocks***
- ***Look at 6-21 a un-replicated experiment and the results***

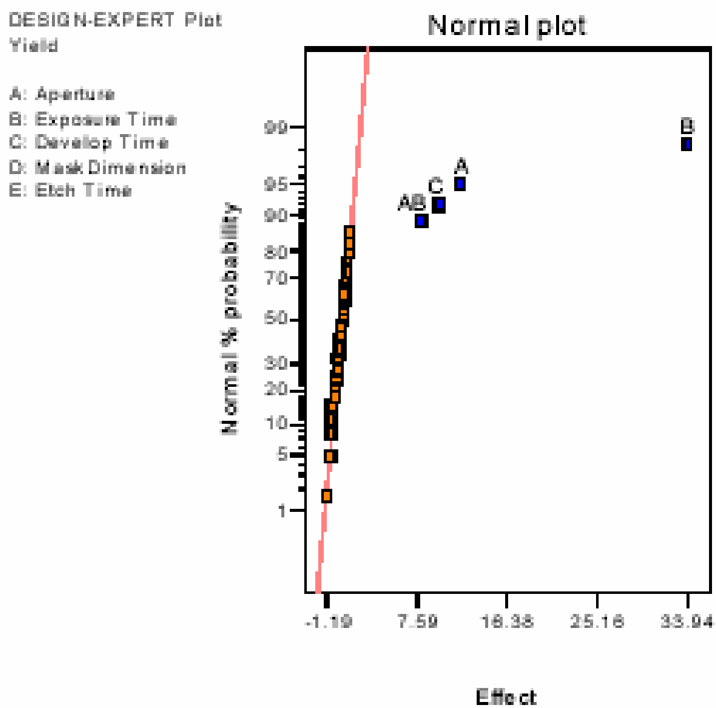
6-21 An experiment was run in a semiconductor fabrication plant in an effort to increase yield. Five factors, each at two levels, were studied. The factors (and levels) were A = aperture setting (small, large), B = exposure time (20% below nominal, 20% above nominal), C = development time (30 s, 45 s), D = mask dimension (small, large), and E = etch time (14.5 min, 15.5 min). The unreplicated 2^5 design shown below was run.

(1) =	7	*d* =	8	*e* =	8	*de* =	6
a =	9	*ad* =	10	*ae* =	12	*ade* =	10
b =	34	*bd* =	32	*be* =	35	*bde* =	30
ab =	55	*abd* =	50	*abe* =	52	*abde* =	53
c =	16	*cd* =	18	*ce* =	15	*cde* =	15
ac =	20	*acd* =	21	*ace* =	22	*acde* =	20
bc =	40	*bcd* =	44	*bce* =	45	*bcde* =	41
abc =	60	*abcd* =	61	*abce* =	65	*abcde* =	63

Chapter 7 Examples

- ***Problem: 7-9 setup a 2^5 in 4 blocks***
- ***Look at 6-21 a un-replicated experiment and the***

(a) Construct a normal probability plot of the effect estimates. Which effects appear to be large?



Chapter 7 Examples

- ***Problem: 7-9 Blocked 6-21 experiment 2^5 in 4 blocks. Conclusions?***

Response: Yield

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source		Sum of Squares	DF	Mean Square	F Value	Prob > F	
Block		2.59	3	0.86			
Model		11585.13	4	2896.28	911.62	< 0.0001	significant
	A	*1116.28*	*1*	*1116.28*	*351.35*	*< 0.0001*	
	B	*9214.03*	*1*	*9214.03*	*2900.15*	*< 0.0001*	
	C	*750.78*	*1*	*750.78*	*236.31*	*< 0.0001*	
	AB	*504.03*	*1*	*504.03*	*158.65*	*< 0.0001*	
Residual		76.25	24	3.18			
Cor Total		11663.97	31				

Chapter 7 Examples

- ***Problem: 7-9 No Blocking 6-21 experiment 2^5 in 4 blocks. Conclusions?***

Analysis of variance table [Partial sum of squares]

Source		Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model		11585.13	4	2896.28	991.83	< 0.0001	significant
	A	*1116.28*	*1*	*1116.28*	*382.27*	*< 0.0001*	
	B	*9214.03*	*1*	*9214.03*	*3155.34*	*< 0.0001*	
	C	*750.78*	*1*	*750.78*	*257.10*	*< 0.0001*	
	AB	*504.03*	*1*	*504.03*	*172.61*	*< 0.0001*	
Residual		78.84	27	2.92			
Cor Total		11663.97	31				

Chapter 7 Examples

- ***Problem: 7-9 Blocked 6-21 experiment 2^5 in 4 blocks***

Std. Dev.	1.78	R-Squared	0.9935
Mean	30.53	Adj R-Squared	0.9924
C.V.	5.84	Pred R-Squared	0.9884
PRESS	135.56	Adeq Precision	63.046

- ***Problem: 7-9 Un-Blocked 6-21 experiment 2^5***

Std. Dev.	1.71	R-Squared	0.9932
Mean	30.53	Adj R-Squared	0.9922
C.V.	5.60	Pred R-Squared	0.9905
PRESS	110.75	Adeq Precision	82.071