Basic Statistics Sample size?

- **Sample size determination: text section 2-4-2 Page 41**
- **section 3-7 Page 107**
- **Website::*http://www.stat.uiowa.edu/~rlenth/Power/***

• Explanation on how to use to develop a sample plan or Answer question: "How many samples should I take to ensure confidence in my data?"

•***Basics:***

•To detect Large Difference in sample populations = small sample size

•To detect Small Difference in sample populations = Large sample size

•The Sample Size required is a function of σ estimate.

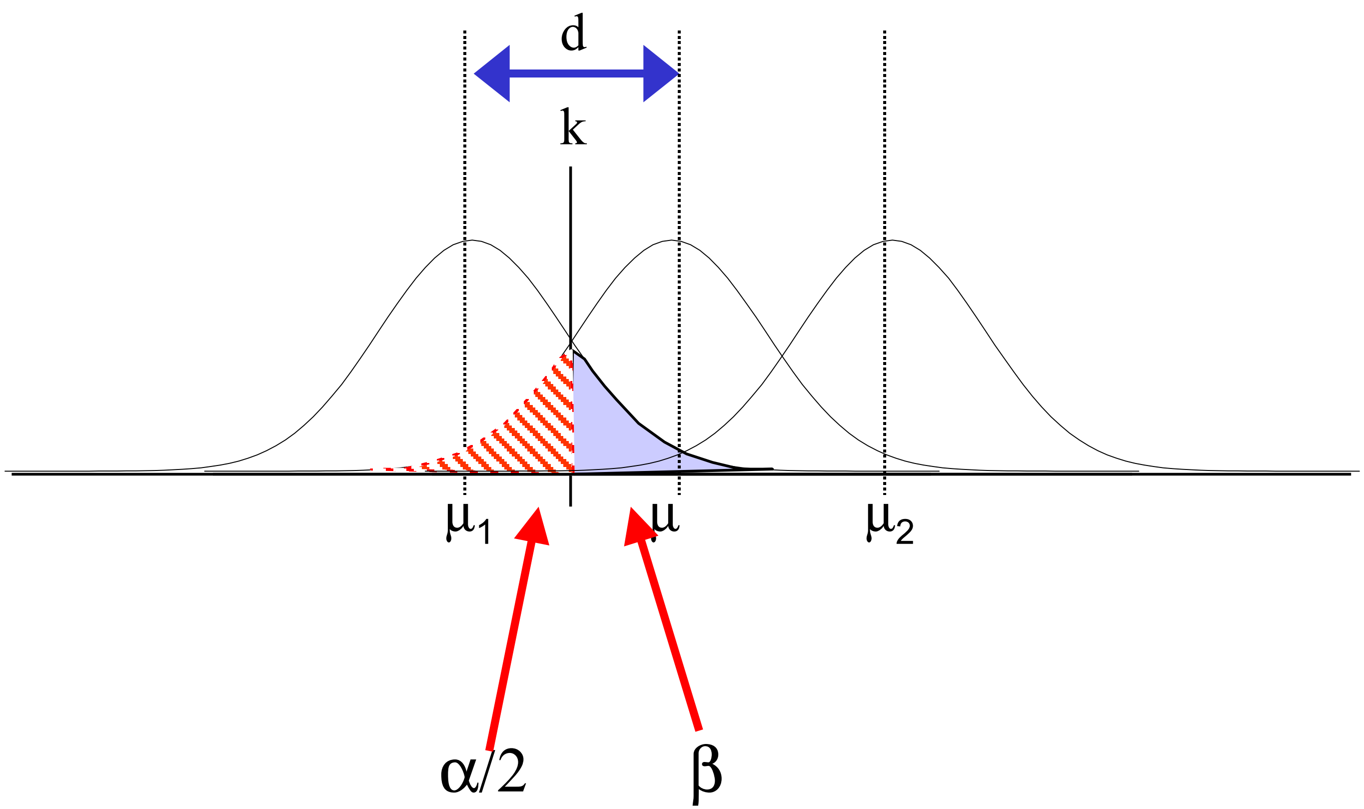
•Sample size is also a function of alpha and beta risks.

•Alpha risk α : It is the risk of wrongly rejecting the null hypothesis H_o, when it is true.

•Beta risk β : It is the risk of wrongly accepting the null hypothesis H_o, when it is false.

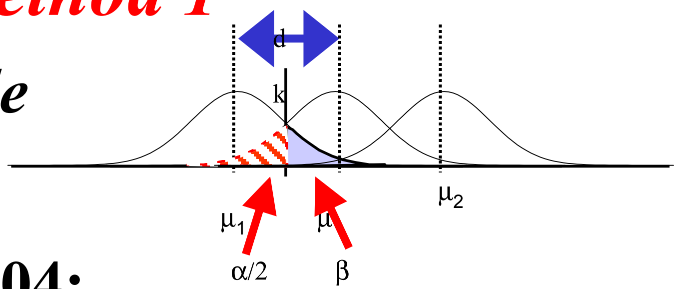
Basic Statistics Sample size? Method 1

- **Given a Process Standard Deviation, What value of n will be enable us to find a specified difference (d) in the means of two populations with α and β defined?**
  - **Define k as left = alpha and right = beta risks**



Basic Statistics Sample size? *Method 1*

- ***Method 1: Example***



- **Example Let: μ = 1.00; σ = 0.07 ; d = 0.04;**
- **$\alpha/2$ = 0.025; β = 0.10**
  - **Define expression for t value of the portion of k with reference to mean μ**
    - **A.) $t = (k - \mu)/(\sigma/(n)^{0.5})$**
  - **Use alpha risk $\alpha/2$ = (0.025) Z = 0.975**
    - **$1.96 = k - 1.00/(0.07/(n)^{0.5})$**
  - **B.) $t = (k - (\mu + d))/(\sigma/(n)^{0.5})$**
  - **Use beta risk β (0.10) Z = 0.90**
    - **$-1.28 = k - 1.04/(0.07/(n)^{0.5})$**

Basic Statistics Sample size? Method 1

- ***Method 1: Example***
  - **Now we have 2 equations and 2 unknowns:**
  - $1.96 = k - 1.00/(0.07/(n)^{0.5})$
  - $-1.28 = k - 1.04/(0.07/(n)^{0.5})$
  - **Set k and eliminate :**
  - $1.96 + 1.00/(0.07/(n)^{0.5}) = -1.28 + 1.04/(0.07/(n)^{0.5})$
  - $3.24 = 0.04 /(0.07/(n)^{0.5})$
  - $(n)^{0.5} = .07/.012346$
  - $(n)^{0.5} = 5.67$
  - $n = 32.15$
  - **So n sample size = 33**

Basic Statistics Sample size? Method 1

- ***Method 1: example***

- ***Looking closely at this relationship we find Sample size is a function of alpha, beta, difference, and sigma values***

- $\mathbf{n = [(t_{\alpha/2} + t_{\beta}) \sigma/d]^2}$

- $\mathbf{t_{\alpha/2}}$ has opposite sign of $\mathbf{t_{\beta}}$
- This is a two tail alpha risk I.e. just looking for difference, if we were looking for an increase of decrease we would use one tail alpha $\mathbf{t_{\alpha}}$

Basic Statistics Sample size? Method 1

- ***Method 1: example***
  - ***To use***
- $$\mathbf{n = [\ (t_{\alpha/2} + t_{\beta})\ \sigma/d]^2}$$

- ***<u>Requires:</u>***
- 1. Normal populations
- 2. Standard deviations are known and do not change with population shift
- 3. We can specify the difference we want to detect in means
- 4. We can specify the risks we are willing to take α and β

Basic Statistics Sample size? *Method 2*

- ***Method 2: example book page 41-42***
- ***Use plot in text figure 2-12 Probability of Accepting Null (Type II error (beta)) Vs d***
- ***Requires:***
- 1. Normal populations
- 2. Standard deviations are know and do not change with population shift
- 3. ***We need to specify the difference we want to detect in means.***
- 4. The alpha risk α is fixed at 0.05 for this curve
- 5. Sample sizes for the two populations are equal

Basic Statistics Sample size? Method 2

- ***Method 2: example book page 41-42***
- ***Use plot in text figure 2-12 Probability of Accepting Ho Vs d*** ***Define d:***
- $d = | \mu_1 - \mu_2 | / 2\sigma$

- From Curves:
- The greater the difference in the means the smaller the Probability of a type II error for a given sample size n and α.
- As the sample size increases the Probability of a type II error decreases for a given difference in the means and α.

Basic Statistics Sample size? *Method 2*

- ***Method 2: example book page 41-42***
- ***Example 1*** ***using plot in text figure 2-12***
  - $d = | \mu_1 - \mu_2 | / 2\sigma$
- We wish to detect a difference of 2.00 and our standard deviation is 0.5.
  - ***d = 2/(2*0.5) =2***
- We wish to reject the Null 95% of the time: means beta (Probability of acceptance) = 5% or 0.05.
- Look at Curve for Probability of accepting 0.05 and d = 2 then n* = 7
- The curves are for n* = 2n-1
- Sample size n = (n*+1)/2 = (7 + 1) / 2 = 4 = $n_1 = n_2$ sample sizes
- ***So need to take 4 samples from n_1 and 4 from n_2 !***

Basic Statistics Sample size? Method 2

- ***Method 2: example book page 41-42***
- ***Example 2 using plot in text figure 2-12***
  - $d = | \mu_1 - \mu_2|/2\sigma$
- We wish to detect a difference of 0.5 and our standard deviation is 0.5.
  - ***d = 0.5/(2*0.5) =0.5***
- We wish to reject the Null 95% of the time: means beta (Probability of acceptance) = 5% or 0.05.
- Look at Curve for Probability of accepting 0.05 and d = 2 then n* = 75
- The curves are for n* = 2n-1
- Sample size n = (n*+1)/2 = (75 + 1) / 2 = 38 = $n_1 = n_2$ sample sizes
- ***So need to take 38 samples from n_1 and 38 from n_2!***

Sample Size and Statistical Risks

- ***Power of test: 1 – β***: Large differences δ require small sample sizes to detect, while small differences δ require large sample sizes!
- Relationship between power 1 – β and sample size *n* (*n* is standardized to 100 for a power of 90%)

1- β	*n*
0.7	59
0.8	75
0.9	100
0.95	123
0.99	175

Sample Size Determination Text, Section 3-7, pg. 107

- **FAQ** in designed experiments
- Answer depends on lots of things; including what type of experiment is being contemplated, how it will be conducted, resources, and desired **sensitivity**
- Sensitivity refers to the **difference in means** that the experimenter wishes to detect
- Generally, **increasing** the number of **replications increases** the **sensitivity** or it makes it easier to detect small differences in means

Sample Size Determination Fixed Effects Case *Method 3*

- Can choose the sample size to detect a specific difference in means and achieve desired values of **type I and type II errors**
- Type I error – reject H_0 when it is true (α)
- Type II error – fail to reject H_0 when it is false (β)
- **Power** = 1 - β
- **Operating characteristic curves** plot β against a parameter Φ where

$$\Phi^2 = \frac{n\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}$$

Basic Statistics Sample size? Method 3

- ***Example book page 107***
- ***Example using plots in text figure V page 647***
- ***Power of test: P = 1-β***

- ***Beta β (Type II error) is*** It is the risk of wrongly accepting the null hypothesis H_o, when it is false. Desire:
  - ***Plot the probability of a type II error β Vs Φ where***

$$\Phi^2 = \frac{n\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}$$

- ***n = sample size or number of replicates ; $\tau = \mu_i - \mu_{bar}$***
- ***a = number of means or variances to compare;***

σ = standard deviation (known)

- ***Example Page 108***

Sample Size Determination
Fixed Effects Case---use of OC Curves
Method 3: example book page 107

- The **OC curves** for the fixed effects model are in the Appendix, Table V, pg. 647
- A very common way to use these charts is to define a difference in two means *D* of interest, then the minimum value of Φ^2 is

$$\Phi^2 = \frac{nD^2}{2a\sigma^2}$$

- Typically work in term of the ratio of D/σ and try values of n until the **desired power** is achieved (i.e. use n to determine β!)
- There are also some other methods discussed in the text

Sample Size Determination Example *Method 3*

- The **OC curves** for the fixed effects model are in the Appendix, Table V, pg. 647 use $\upsilon_1 = 1$ plot for this example (two means to compare)
- Example **desired power** =0.95 so $\beta = 0.05$
- D = .75
- a = 2 degrees of freedom υ_1 = a-1 =1

  υ_2 error degrees of freedom = a(n-1) = 2(n-1)

  $\alpha = 0.05$

  $\sigma = 0.5$
- Calculate $\Phi^2 = \frac{nD^2}{2a\sigma^2}$ = n0.5625/2*2*.25)=2.25n

Sample Size Determination Example *Method 3*

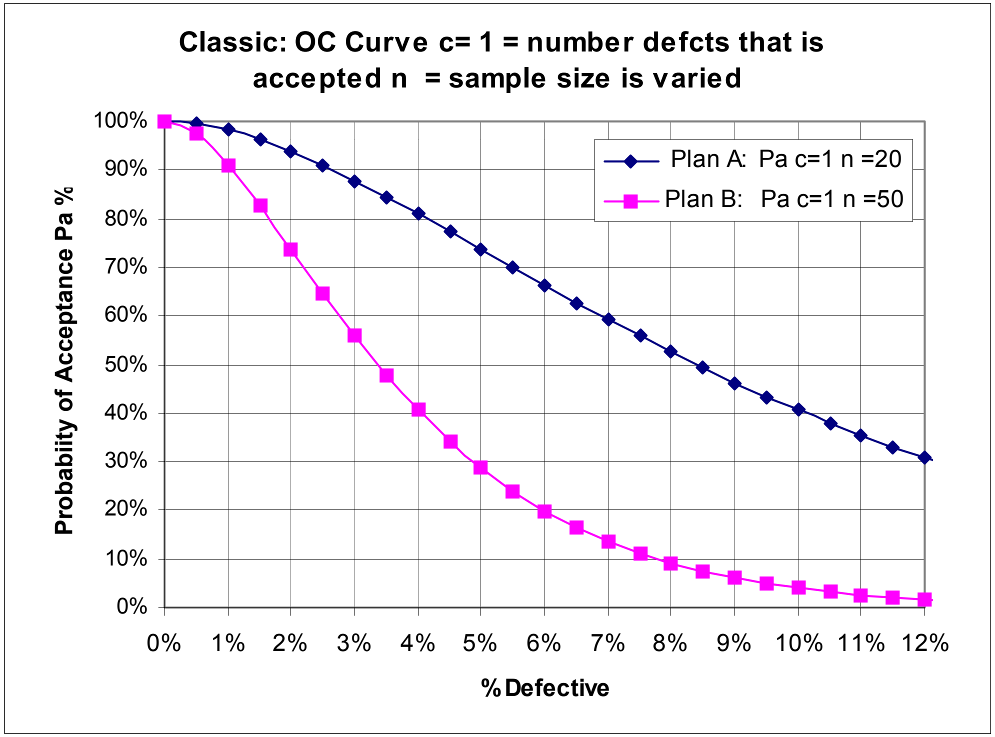
- The **OC curves** for the fixed effects model are in the Appendix, Table V, pg. 647 use $\upsilon_1 = 1$ plot for this example (2 means to compare)

$\Phi^2 = 2{,}25n$ α = ***0,05***

n Replicates	Φ^2	Φ	υ_2 *error degrees of freedom = a(n-1) = 2(n-1)*	*beta from plot page 247*
1	2.25	1.50	0	
2	4.5	2.12	2	
3	6.75	2.60	4	
4	**9**	**3.00**	**6**	**0.06**
5	**11.3**	**3.35**	**8**	**0.02**
6	13.5	**3.67**	**10**	
7	15.75	3.97	12	

Basic Statistics OC Curve Acceptance Sampling

- **THE OPERATING CHARACTERISTIC CURVE (OC CURVE)**
- The Operating characteristic curve is a picture of a sampling plan. Each sampling plan has a unique OC curve. The sample size and acceptance number define the OC curve and determine its shape. ***The acceptance number is the maximum allowable defects or defective parts in a sample for the lot to be accepted.*** The OC curve shows the probability of acceptance for various values of incoming quality.

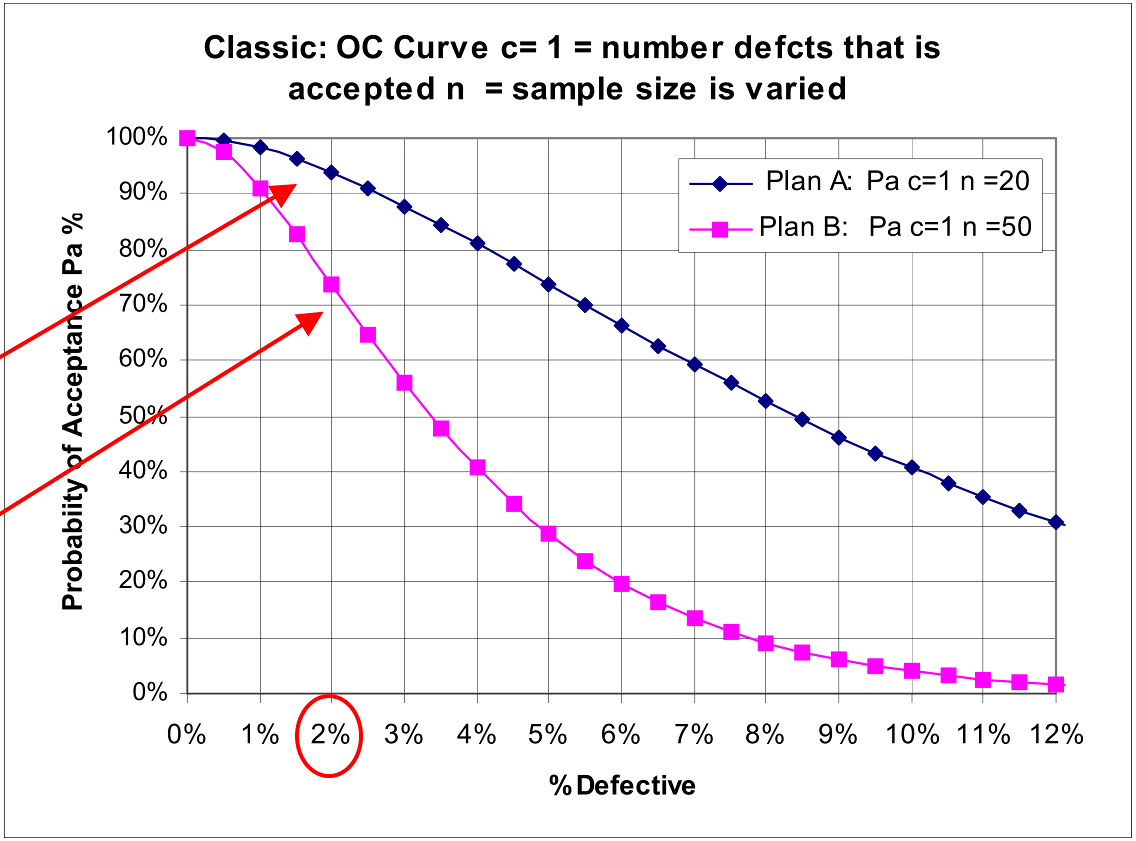


% defective = p; P(a) = Prob, of acceptance (b); n = sample size c = # of defects will to accept

- **Explanation on how to use to develop a sample plan or Answer question: "How many samples should I take to ensure confidence in my data?"**

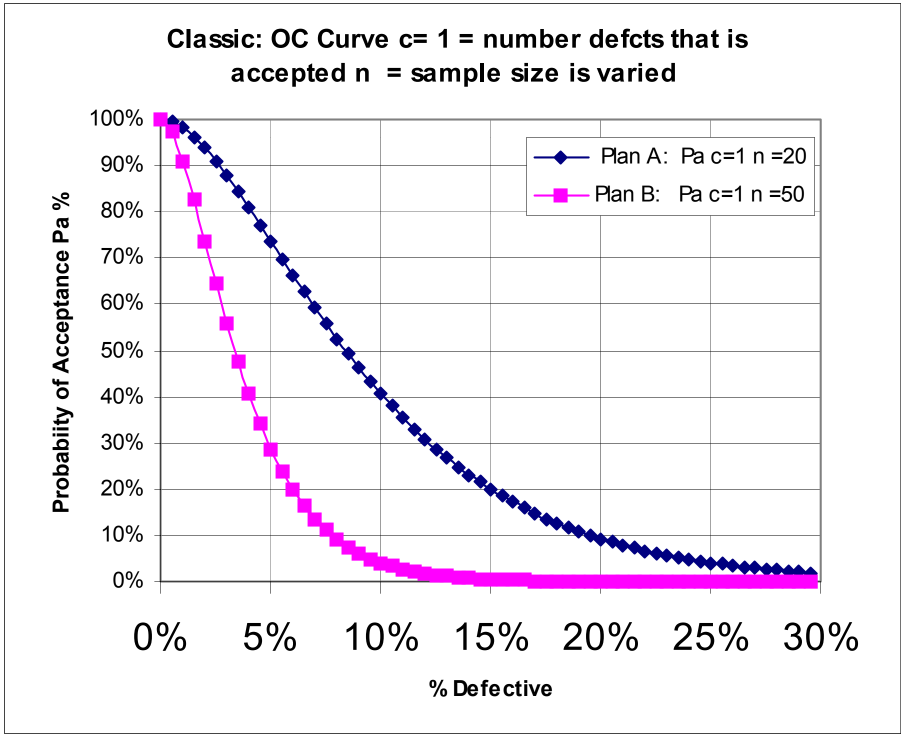
Basic Statistics OC Curve Acceptance Sampling

- Two sample plans A & B:
- Both inspection plans call for accepting the lot based on finding only ***one defective part.***
- If there are 2% defective parts in my population :
- **Plan A (n=20):** I have a probability of 93% of accepting the lot.
- **Plan B (n=50):** : I have a probability of 73% of accepting the lot.
- ***Further:Acceptance***
- PLAN A: 50:50 chance accepting 8.1% defective
- PLAN B: 50:50 chance accepting 3.3% defective



Basic Statistics OC Curve Acceptance Sampling

- Two sample plans A & B:
- Further:Acceptance
- ***PLAN A (n=20):*** 10% chance of accepting 20% defective
- ***PLAN B (n=50):*** 10% chance of accepting 7.6% defective
- Does not sound too good!
- How do I improve:
- Increase n more = $$.
- If you increase c (number of defects in sample you are willing to accept) chance of accepting increases!



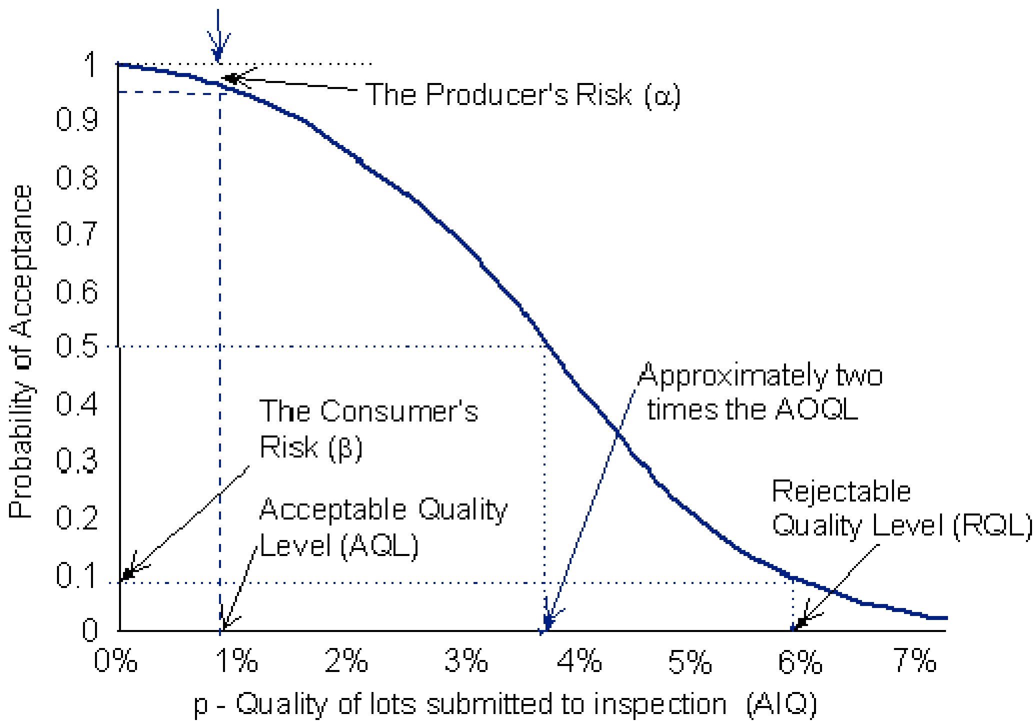
Basic Statistics OC Curve Acceptance Sampling

Producers risk: Probability or risk of rejecting the product when it is good:

EXAMPLE: If process runs normally at 1% defective, probability of acceptance is 95%. Producers risk is 1 – 0.95 – 0.05 or 5% !

Consumers risk: Probability or risk of accepting the product when it is bad:

EXAMPLE: Define a level at which the product consumer wants the product rejected at 6% defective, probability of acceptance is 10%. Consumers risk is 10% for the defined Percent defective !



AQL : Defined so there is a high probability of acceptance

RQL : Defined so there is a low probability of acceptance

OC Curve Acceptance Sampling: Vary # defects accepted

- **OC Curves: Operating Characteristic Curves**

Generated using the Poisson formula

n = sample size

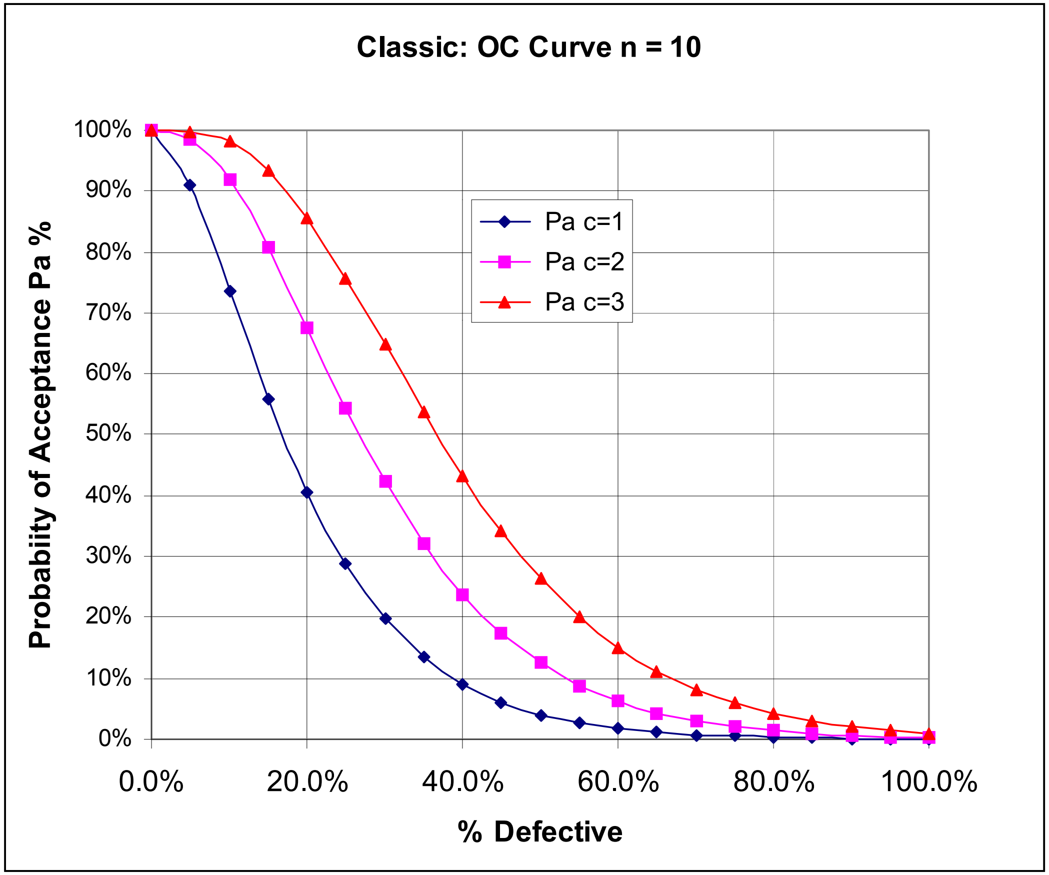
p = % defective in population or % difference you want to detect

np = λ = # expected defects in sample

x = c = number of defects found in the sample = *acceptance Number.*

If P(x) or Pa = probability of acceptance

$$P(x) = \frac{e^{-np}(np)^x}{x!}$$



OC Curve construction Acceptance Sampling

P(x) or P(a) = probability of acceptance

Y axis calculated from Poisson distribution

Poisson Distribution

$$P(x) = \frac{e^{-np}(np)^x}{x!}$$

n = sample size

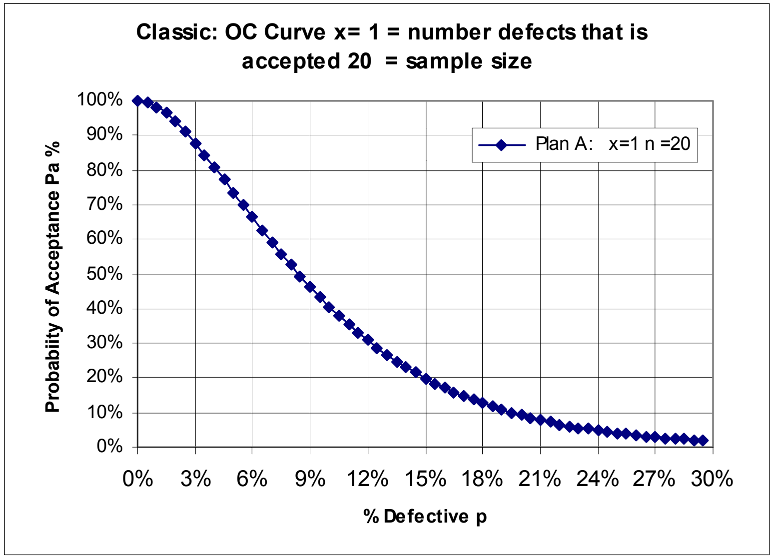
Example = 20

x =1= number of defects found in the sample that you are willing to accept = *acceptance Number.*

p = % defective in population (known from past experience)

X axis (Varied)

np = λ = # expected defects in sample



OC Curve construction Acceptance Sampling

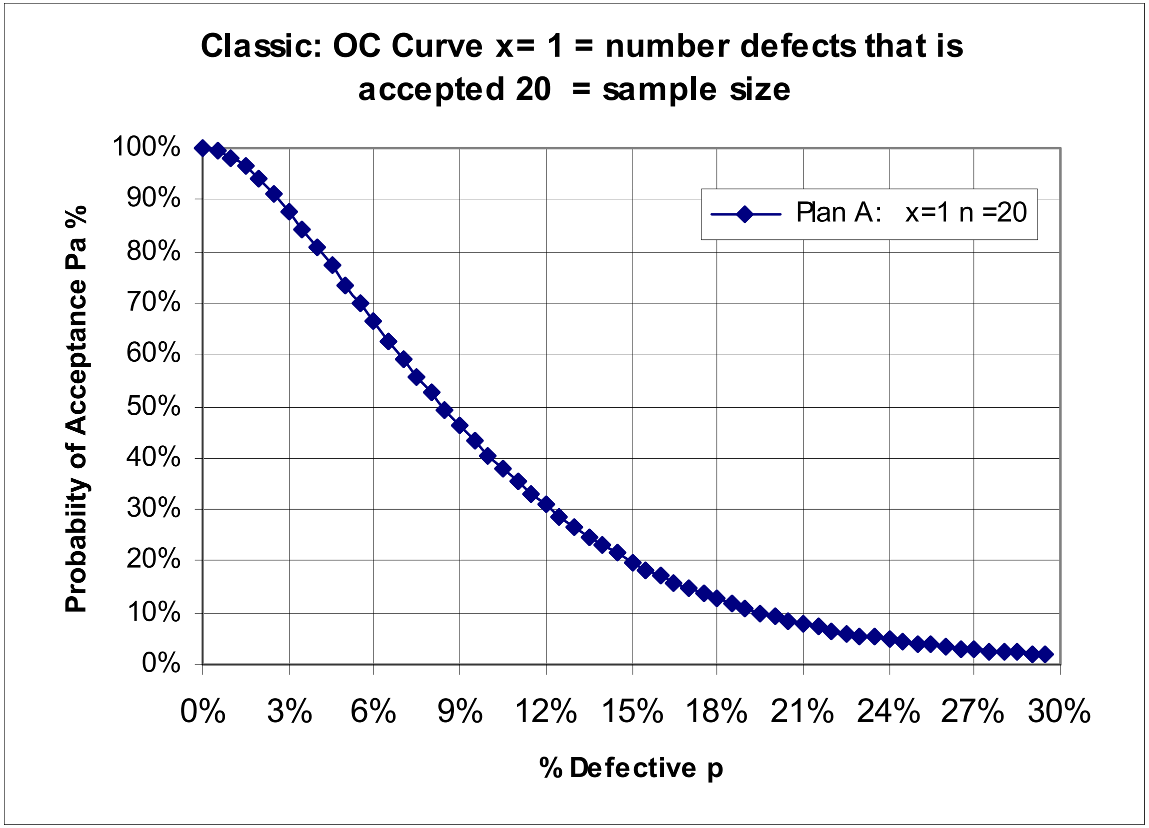
P(x) or P(a) = probability of acceptance

Y axis calculated from Poisson distribution

So with this sample plan:

I accept the lot if I find only 1 defect in sample and reject the lot if I find more than 1 defect.

With this sampling plan I have a 30% probability of accepting the lot, if the real population has 12% defective parts.

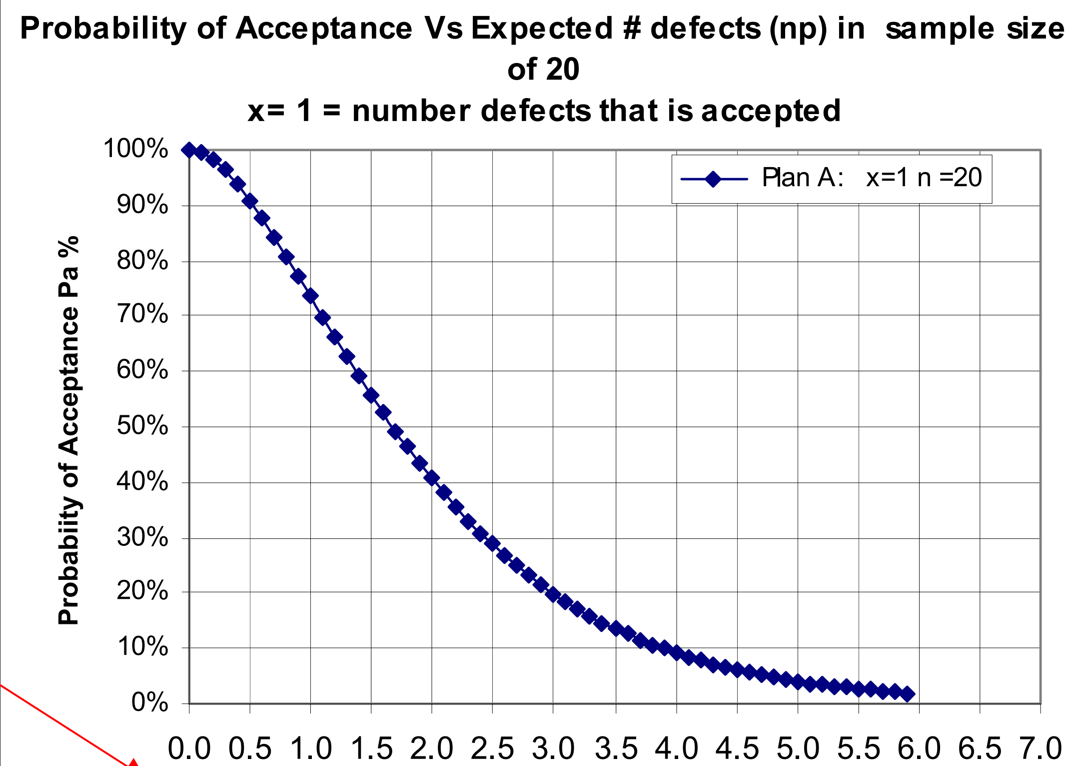


OC Curve construction Acceptance Sampling expected # defects in sampel size 20 for varying % defective levels P

P(x) = probability of acceptance

Y axis calculated from Poisson distribution

Let p vary



np = λ = # expected defects in sample 20 *p

OC Curve Acceptance Sampling: Vary Sample size n

- **OC Curves: Operating Characteristic Curves Examples**

Generated using the Poisson formula

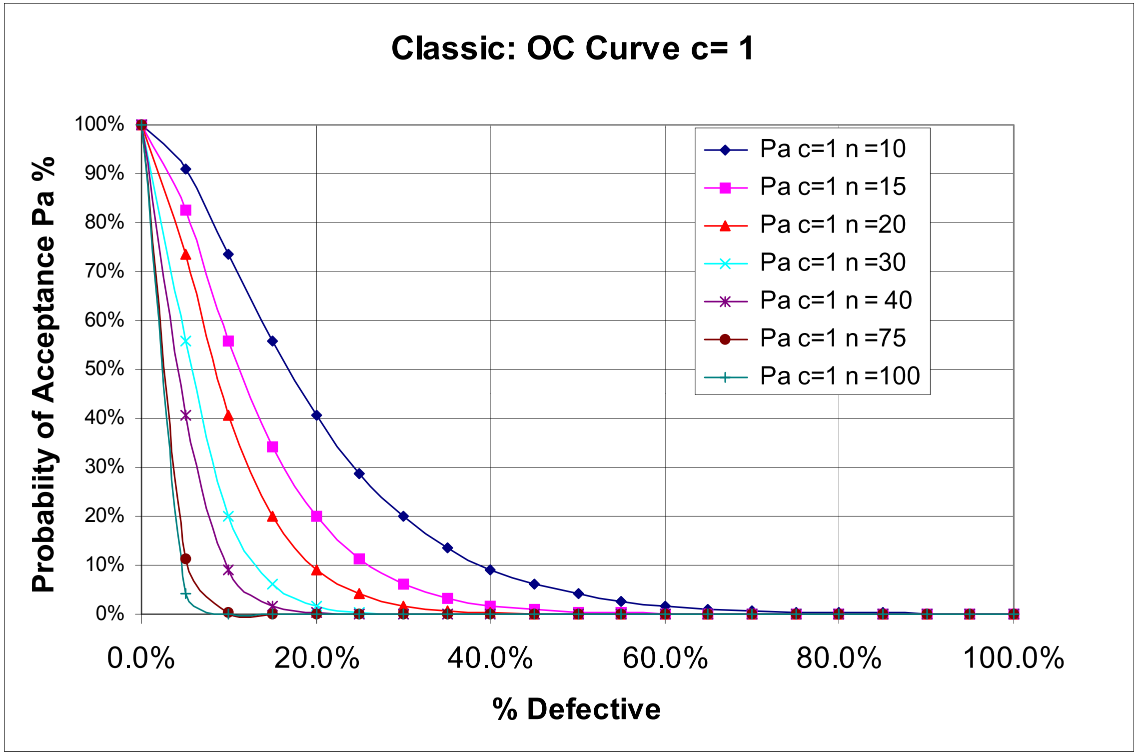
n = sample size

p = % defective in population or % difference you want to detect

x = c = acceptance number number of defects found in the sample. If

P(x) or Pa = probability of acceptance

$$P(x) = \frac{e^{-np}(np)^x}{x!}$$



Basic Statistics OC Curve Acceptance Sampling

- **THE OPERATING CHARACTERISTIC CURVE (OC CURVE) Generation**
- An OC curve is developed by determining the probability of acceptance for several values of incoming quality. Incoming quality is denoted by p. The probability of acceptance is the probability that the number of defects or defective units in the sample is equal to or less than the acceptance number of the sampling plan. The AQL is the acceptable quality level and the RQL is rejectable quality level. If the units on the abscissa are in terms of percent defective, the RQL is called the LTPD or lot tolerance percent defective. ***The producer's risk (α) is the probability of rejecting a lot of AQL quality. The consumer's risk (β) is the probability of accepting a lot of RQL quality.***
- There are three probability distributions that may be used to find the probability of acceptance. These distributions are.
- · **The hypergeometric distribution**
- · **The binomial distribution**
- · **The Poisson distribution**
- Although the hypergeometric may be used when the lot sizes are small, the ***binomial and Poisson are by far the most popular distributions to use when constructing sampling plans.***