Design of Engineering Experiments Chapter 6 – Full Factorial Example

- Example worked out Replicated Full Factorial Design

- 2^3 Pilot Plant : Response: % Chemical Yield:

- If there are **a** levels of Factor A , **b** levels of Factor B, and **c** levels of Factor C a **_full factorial_** design is one in all abc combinations are tested. When factors are arranged in a factorial design, they are often called **_crossed_**.

- Model:

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3$$
$$+ b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3$$
$$+ b_{123} x_1 x_2 x_3 + \varepsilon$$

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield Design and data:

- **A**
- **B**
- **C**

FACTORS

			Lo(-1)	Hi(+1)
Temperature			160°	180° C
Concentration			10%	40%
Catalyst			A	B

| Test#	Temp x_1	Conc x_2	Catalyst x_3	Yield y_{i1}	y_{i2}	$\bar{y}$
1	-1	-1	-1	59	61	60
2	+1	-1	-1	74	70	72
3	-1	+1	-1	50	58	54
4	+1	+1	-1	69	67	68
5	-1	-1	+1	50	54	52
6	+1	-1	+1	81	85	83
7	-1	+1	+1	46	44	45
8	+1	+1	+1	79	81	80

Standard Order
- 1st variable column (-1,+1,-1...)
  Alternates every 2^0 value
- 2nd variable column (-1,-1,+,+,-,-,+,+)
  Alternates every 2^1 values
- 3rd variable column (-,-,-,-,+,+,+,+,-,-,-,-,+,+,+,+,...)
  Alternates every 2^2 values

- k^{th} variable - alternates every 2^{k-1} values

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Cubic display: Replicated Experiment
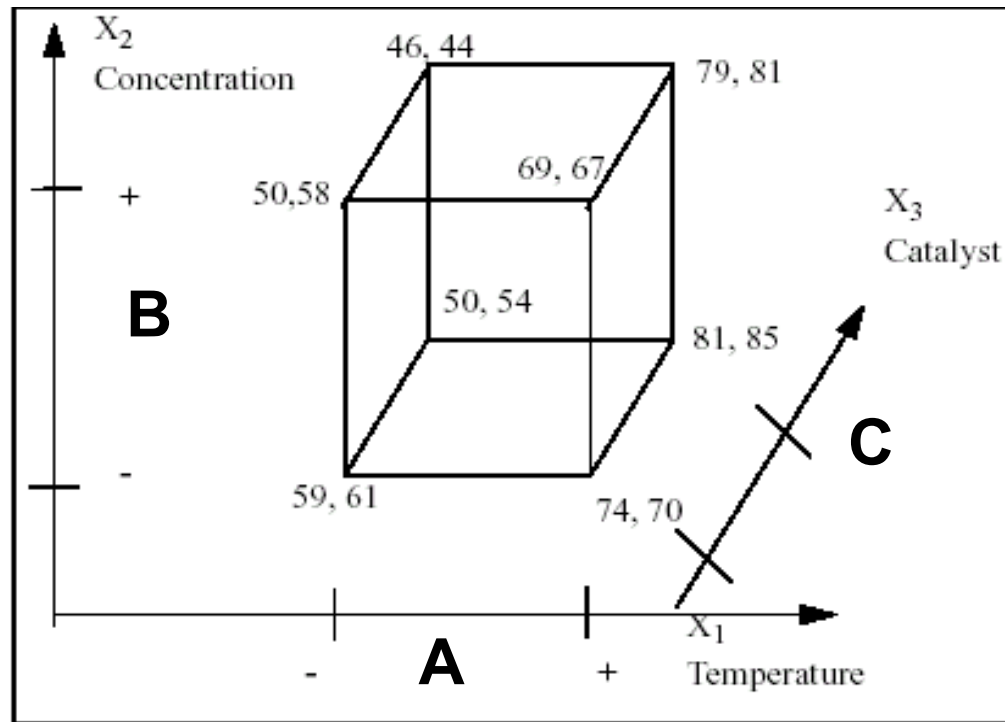
Definition of a factor effect: The change in the mean response when the factor is changed from low to high.

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- The **_effect_** of a factor is defined to be the change in the response Y for a change in the level of that factor. This is called a **_main effect_**, because it refers to the primary factors of interest in the experiment.

- Calculation of Effects:

Avg. of all responses at high level of x_1

$$= \frac{74 + 70 + 69 + 67 + 81 + 85 + 79 + 81}{8} = 75.75$$

Avg. of all responses at low level for x_1

$$= \frac{59 + 61 + 50 + 58 + 50 + 54 + 46 + 44}{8} = 52.75$$

$E_1 = 75.75 - 52.75 = 23.0$ Geometrical interpret

$$E_1 = \frac{-59 - 61 + 74 + 70 - 50 \ldots + 85 - 46 - 44 + 79 + 81}{\text{Number of ''+''Signs} = 8}$$

$E_1 = 23.0$

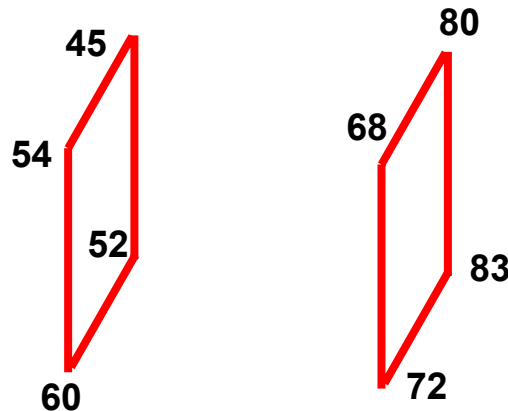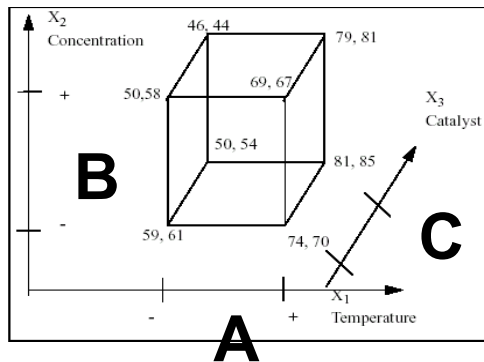$$E_1 = \frac{-60 + 72 - 54 + 68 - 52 + 83 - 45 + 80}{4} = 23.0$$

$\bar{y}$'s (We will work with these)

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***A Main effect***

Effect of A: Average of all the High A's
minus the average of all the Low A's

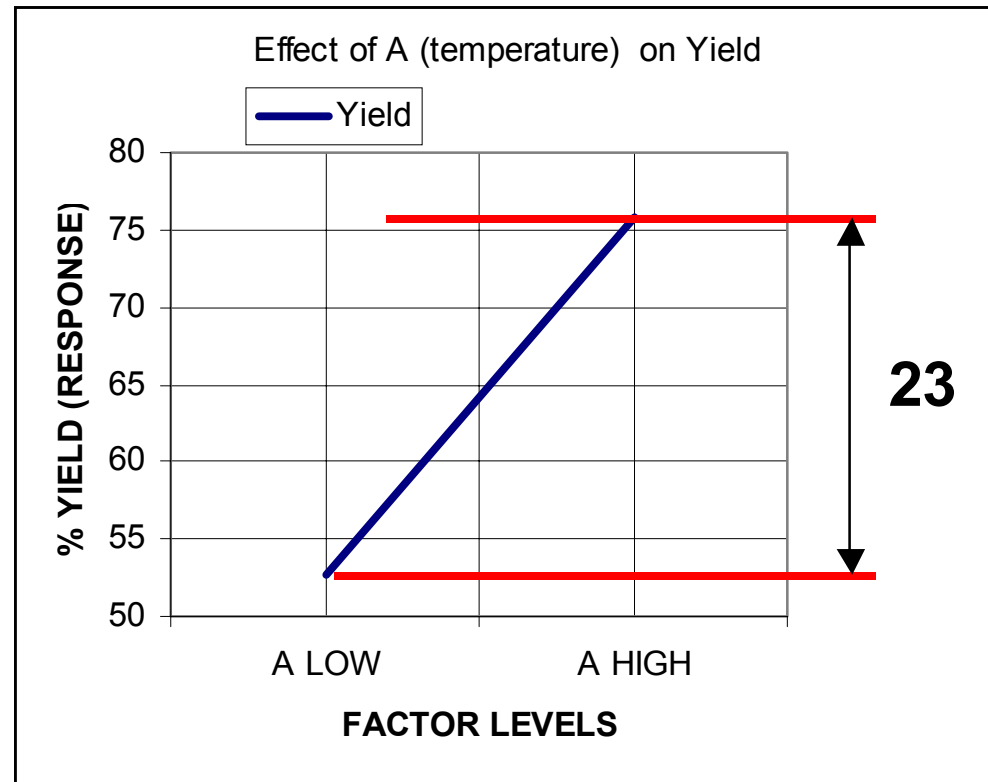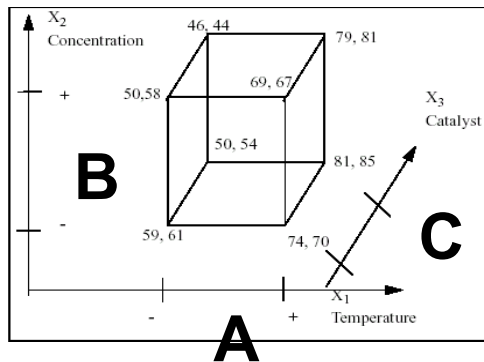|	High A	Low A	Effect A
	68	54	
	72	60	
	83	52	
	80	45	
Average	75.75	52.75	**23**

low A   high A

Effect of A = E_A = (68+72+83+80)/4 – (54+60 +52+45)/4 = 23

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***A Main effect***

Effect of A: Average of all the High A's minus the average of all the Low A's
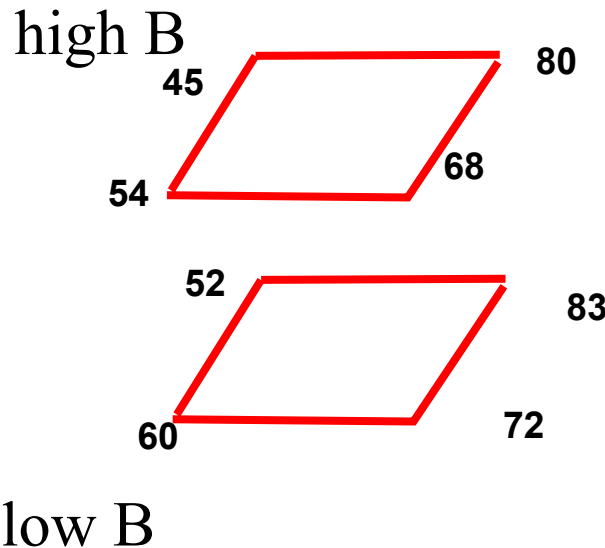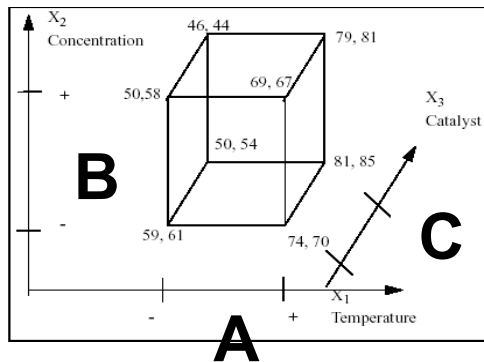

Effect of A (temperature) on Yield

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***B Main effect***

Effect of B: Average of all the High B's minus the average of all the Low B's

high B

low B

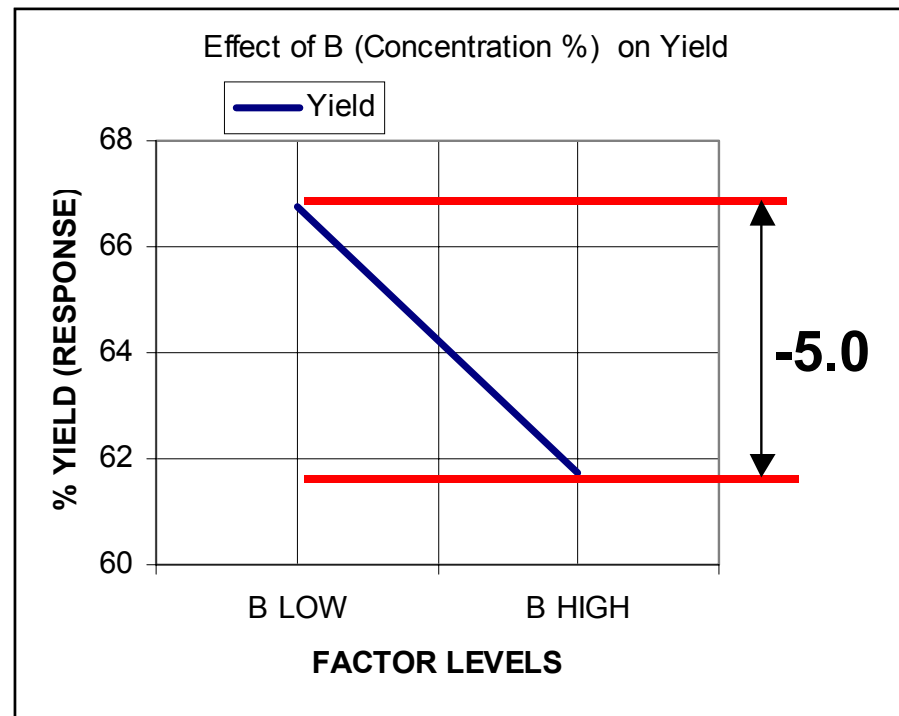|	High B	Low B	Effect B
	68	72	
	54	60	
	45	52	
	80	83	
Average	61.75	66.75	**-5**

Effect of B = E_B = (68+54+45+80)/4 – (72+60 +52+83)/4 = -5

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: **_B Main effect_**

  Effect of B: Average of all the High B's minus the average of all the Low B's
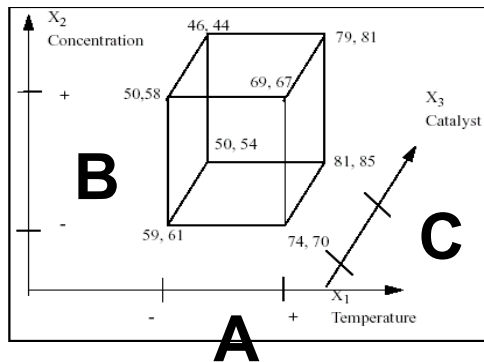
B

C

A

Effect of B (Concentration %) on Yield

Yield

-5.0

% YIELD (RESPONSE)

B LOW   B HIGH

FACTOR LEVELS

Design of Engineering Experiments
Chapter 6– Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***C Main effect***

Effect of C: Average of all the High C's
minus the average of all the Low C's

	High C	Low C	Effect C
	45	54	
	52	60	
	83	68	
	80	72	
Average	65	63.5	**1.5**

Effect of C = E_c =(52+54+45+83)/4 – (72+60 +54+68)/4 = 1.5

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***C Main effect***

Effect of C: Average of all the High C's minus the average of all the Low C's

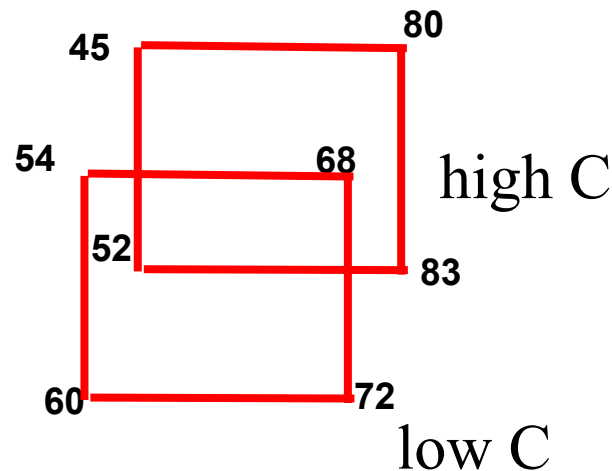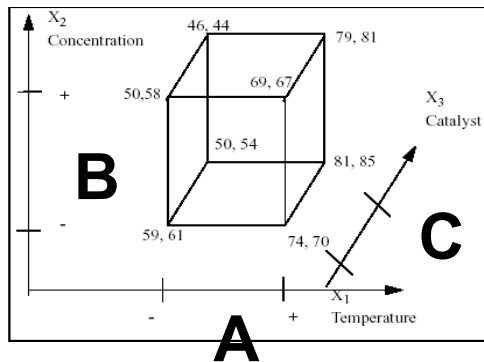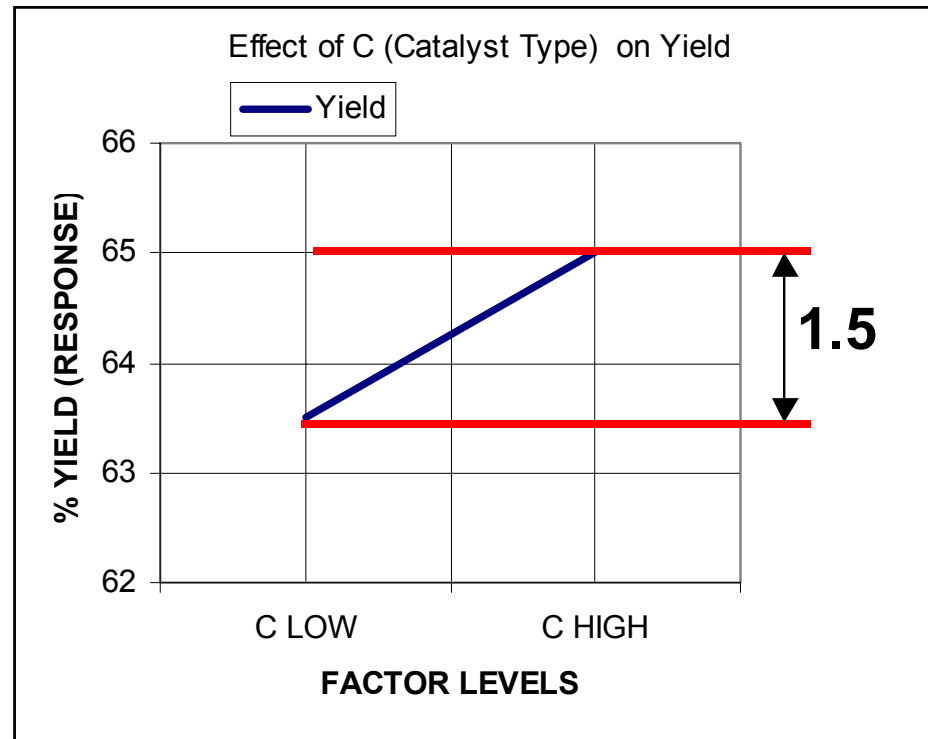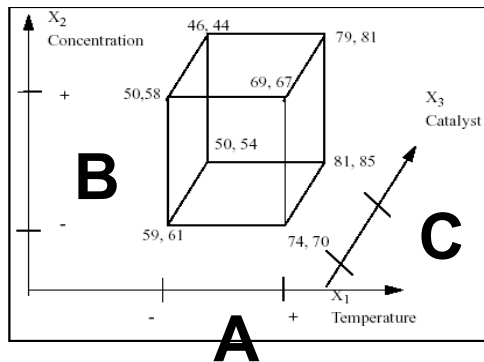Effect of C (Catalyst Type) on Yield

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***AB Interaction***

Effect of AB: Average of all the positive A*B's plus the average of all the negative A*B's

AB	+	AB	-	Effect AB
- -	60	+ -	72	
+ +	68	- +	54	
- -	52	+ -	83	
+ +	80	- +	45	
Average	65		63.5	**1.5**

Effect of AB = E_{AB} = (60+68+52+80)/4 – (72+54 +83+45)/4 = 1.5

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***AB Interaction effect***

   Effect of AB: Average of all the positive A*B's plus the average of all the negative A*B's
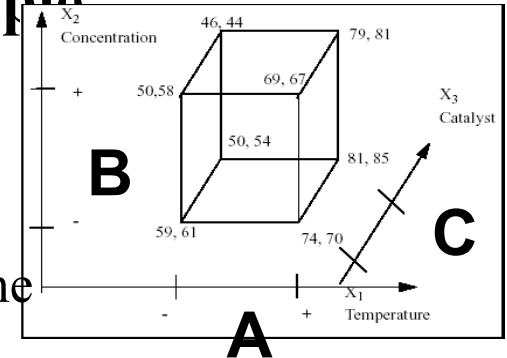
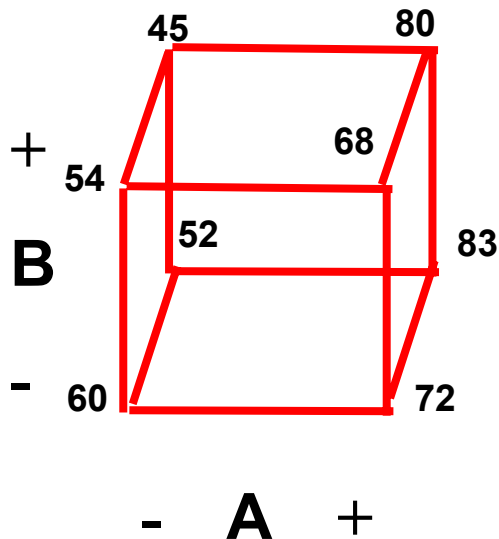| FACTOR AB	YIELD B LOW 1 (C LOW)	YIELD B LOW 2 (C HIGH)	AVG Low B	YIELD B HIGH 1 (C LOW)	YIELD B HIGH 2 (C HIGH)	AVG High B	AB AVERAGE
AB LOW	60	52	**56**	54	45	**49.5**	**63.5**
AB HIGH	72	83	**77.5**	68	80	**74**	**65**
						EFFECT	**1.5**

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***AB Interaction effect***

  Effect of AB: Average of all the positive A*B's plus the average of all the negative A*B's

Effect of AB (Temperature XConcentration Interaction) on Yield:
Average of all the positive A*B's plus the average of all the negative A*B's

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***AC Interaction***

Effect of AC: Average of all the positive A*C's plus the average of all the negative A*C's

45 80

68

54

52 83 **+**

C

60 72

-

- A +

AC	+	AC	-	Effect AC
- -	60	+ -	72	
+ +	80	- +	45	
- -	54	+ -	68	
+ +	83	- +	52	
Average	69.25		59.25	**10**

Effect of AC = E_{AC} = (60+80+54+83)/4 – (72+45 +68+52)/4 = 10

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***AC Interaction effect***

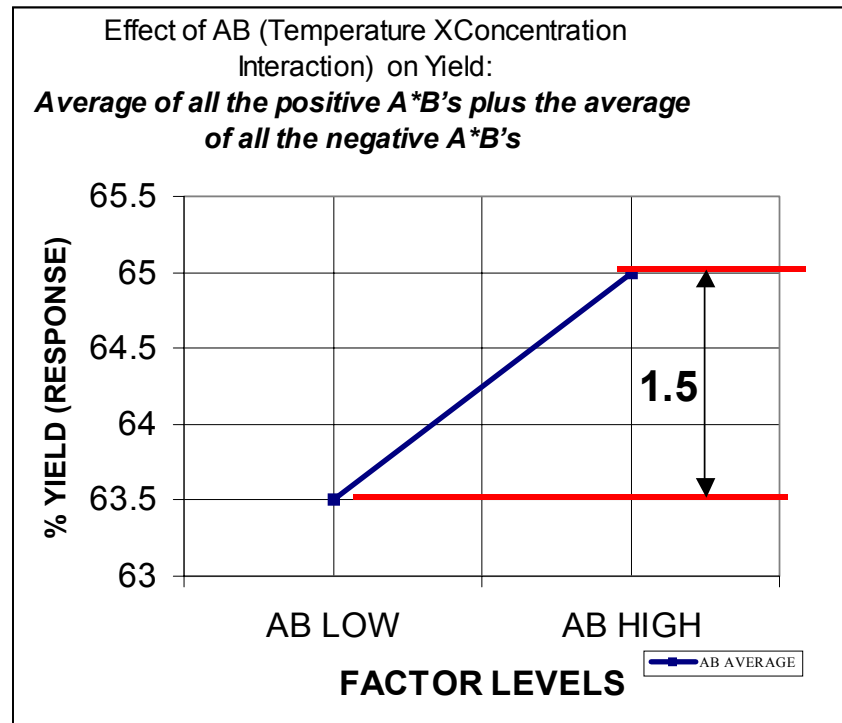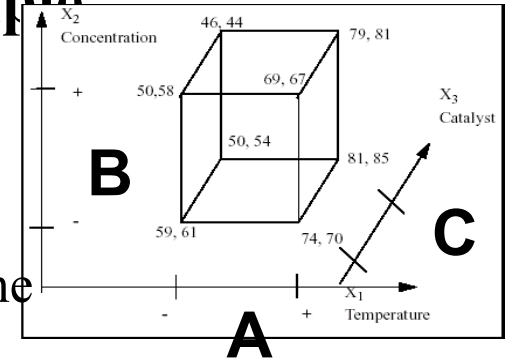   Effect of AC: Average of all the positive A*C's plus the average of all the negative A*C's
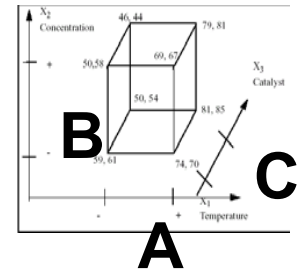
FACTOR AC	YIELD C LOW 1 (B LOW)	YIELD C LOW 2 (B HIGH)	AVG Low C	YIELD C HIGH 1 (B LOW)	YIELD C HIGH 2 (B HIGH)	AVG High C	AC AVERAGE
AC LOW	60	54	**57**	52	45	**48.5**	**59.25**
AC HIGH	72	68	**70**	83	80	**81.5**	**69.25**
						EFFECT	**10**

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***AC Interaction effect***

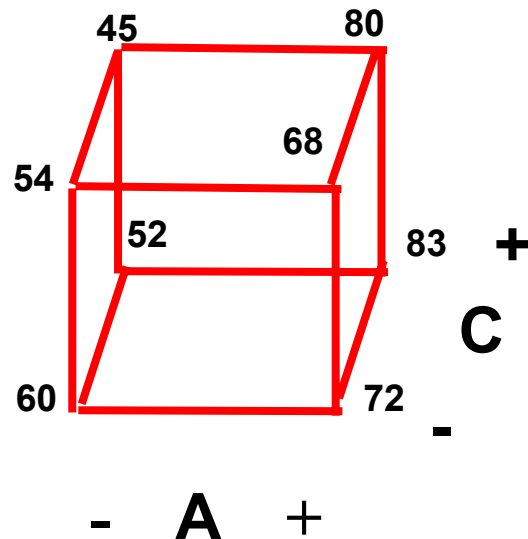 Effect of AC: Average of all the positive A*C's plus the average of all the negative A*C's

Effect of AC (Temperature X Catalyst Interaction) on Yield

Effect of AC (Temperature X Catalyst Interaction) on Yield:
Average of all the positive A*C's plus the average of all the negative A*C's

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***BC Interaction***

Effect of BC: Average of all the positive B*C's plus the average of all the negative B*C's

BC	+	BC	-	Effect BC
- -	60	+ -	54	
+ +	45	- +	52	
- -	72	+ -	68	
+ +	80	- +	83	
Average	64.25		64.25	**0**

Effect of BC = E_{BC} = (60+45+72+80)/4 – (54+52+68+83)/4 = 0

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***BC Interaction effect***

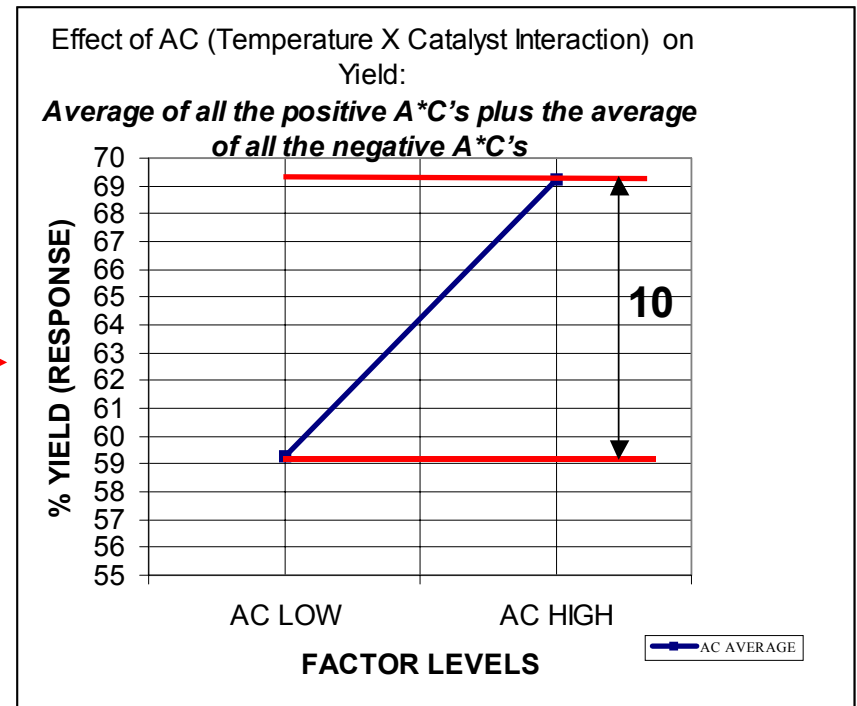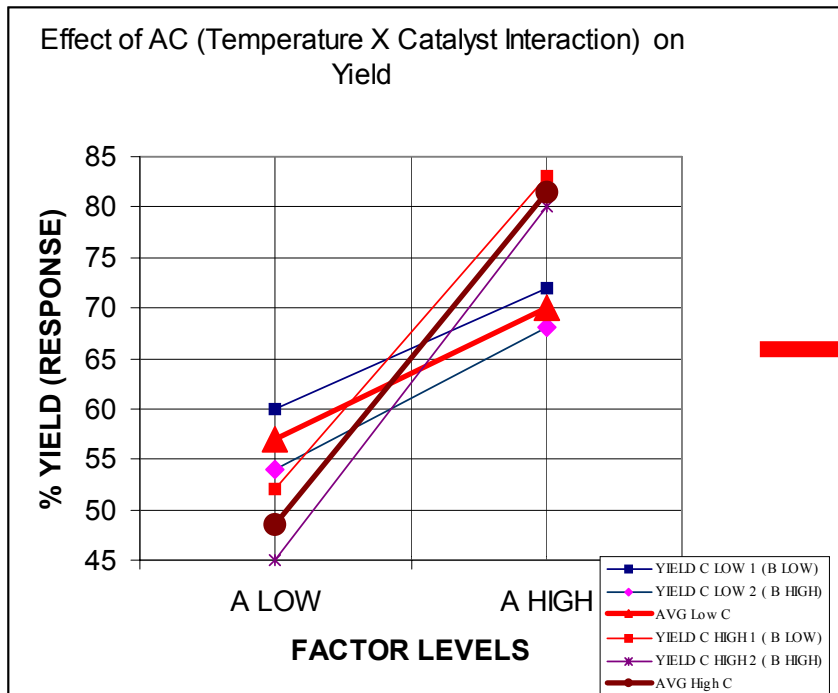Effect of BC: Average of all the positive B*C's plus the average of all the negative B*C's
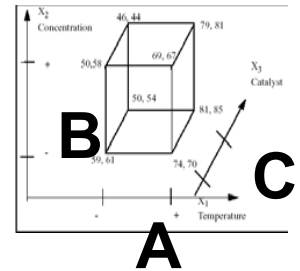
FACTOR BC	YIELD C LOW 1 (A LOW)	YIELD C LOW 2 (A HIGH)	AVG Low C	YIELD C HIGH 1 (A LOW)	YIELD C HIGH 2 (A HIGH)	AVG High C	AC AVERAGE
BC LOW	60	72	**66**	52	83	**67.5**	**64.25**
BC HIGH	54	68	**61**	45	80	**62.5**	**64.25**
						EFFECT	**0**

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2³ Pilot Plant : Response: % Chemical Yield

- Interpretation of effects: ***BC Interaction effect***

Effect of BC: Average of all the positive B*C's plus the average of all the negative B*C's

Effect of AB (Temperature XConcentration Interaction) on Yield:
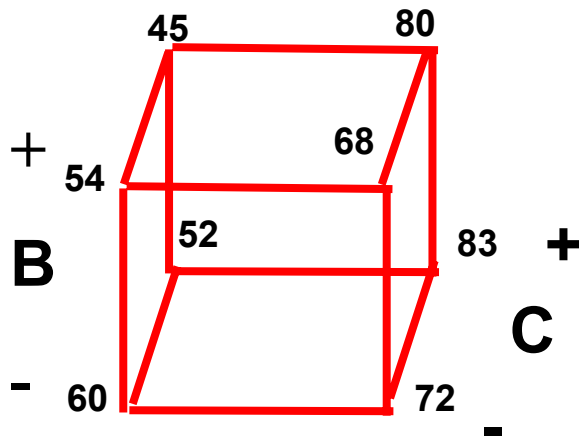***Average of all the positive B*C's plus the average of all the negative B*C's***

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield

- Calculation of Effects: Graphically ***ABC Interaction***

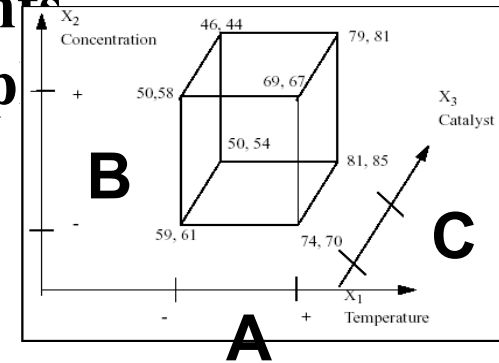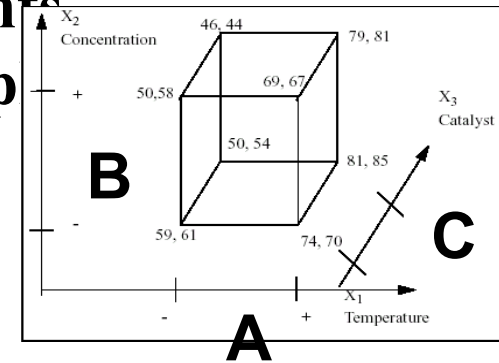Effect of ABC: Average of all the positive A*B*C's plus the average of all the negative A*B*C's
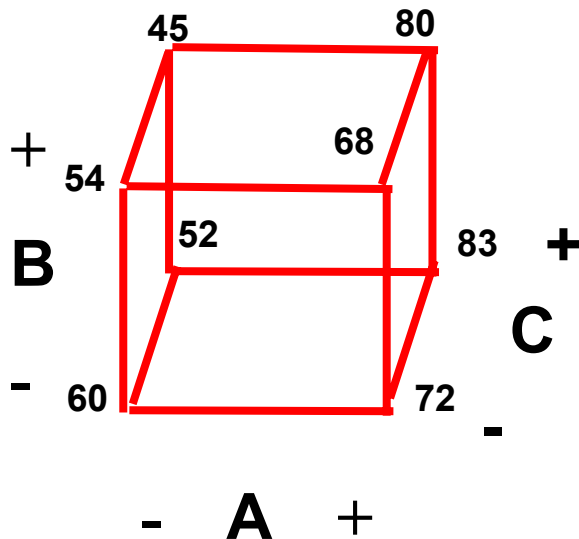
ABC	+	ABC	-	Effect ABC
+ - -	72	- - -	60	
- + -	54	++ -	68	
+ + +	80	- + +	45	
- - +	52	+ - +	83	
Average	64.5		64	**0.5**

Effect of ABC = E_{ABC} = (72+54+80+52)/4 – (60+68 +45+83)/4 = 0.5

Summary: Effects in The 2^3 Factorial Design Geometrical Presentation

(a) Main effects

(b) Two-factor interaction

● = + runs
○ = − runs

(c) Three-factor interaction
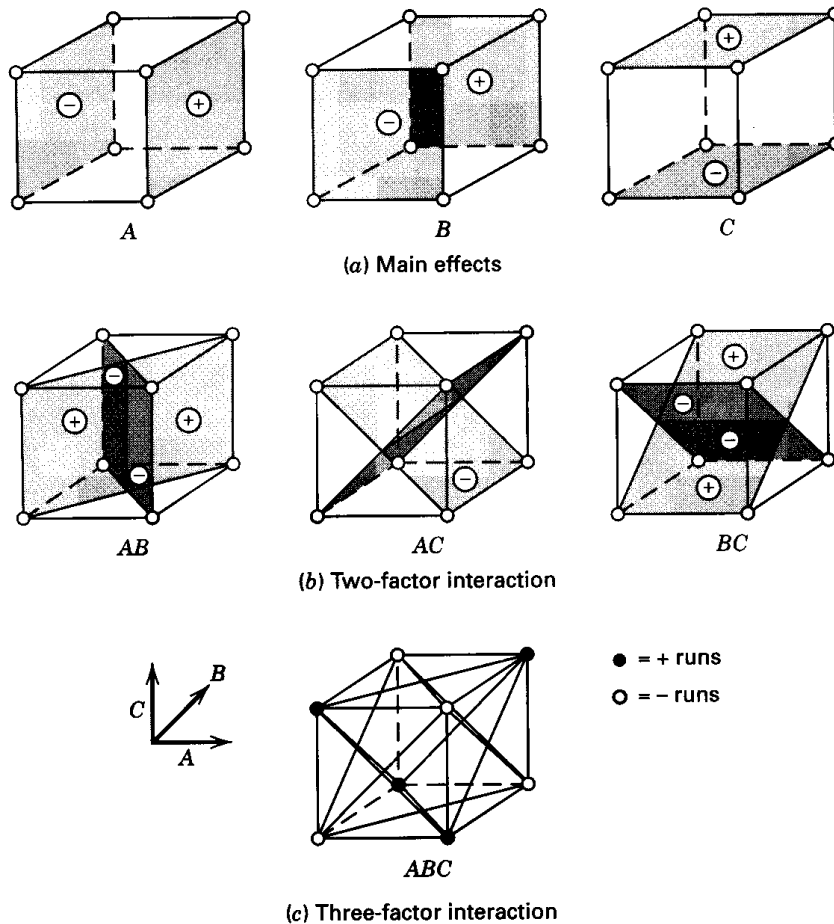
Figure 6-5 Geometric presentation of contrasts corresponding to the main effects and interactions in the 2^3 design.

$$A = \overline{y}_{A^+} - \overline{y}_{A^-}$$

$$B = \overline{y}_{B^+} - \overline{y}_{B^-}$$

$$C = \overline{y}_{C^+} - \overline{y}_{C^-}$$

etc, etc, ...

Thank you for Computers!!

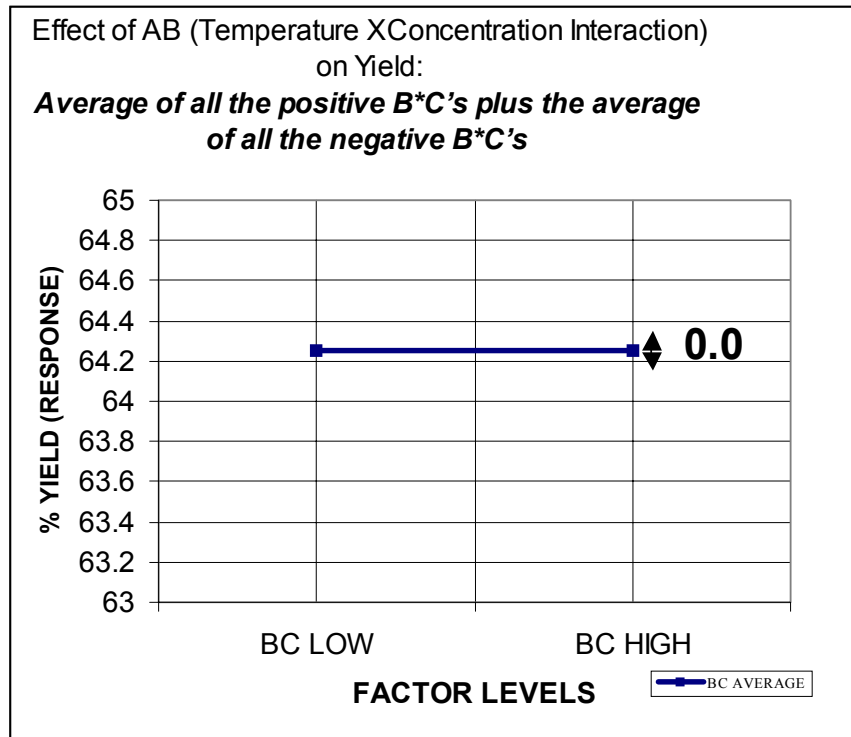Analysis instantly One just needs to know how to input correctly and interrupt results

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Summary of Effects: So Which ones are important??
- I.e Distinguishable from random Noise??
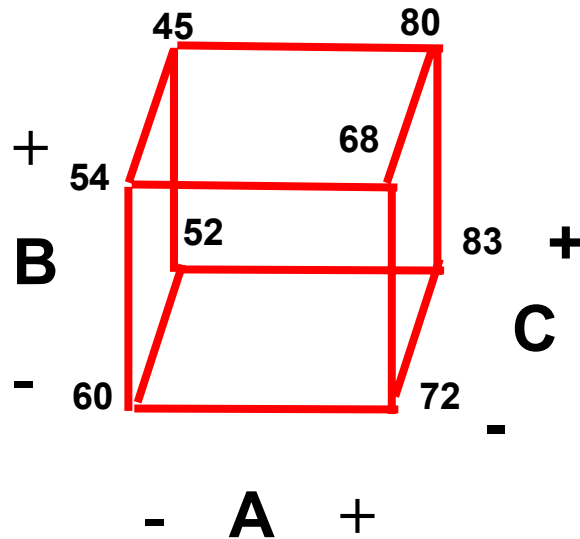
| Effect E_X	Value
A	23
B	-5
C	1.5
AB	1.5
AC	10
BC	0
ABC	0.5

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Summary of Effects: So Which ones are important??
- Experiment was replicated so calculate Sample polled variance as:

Test	y_{i1}	y_{i2}	$\bar{y}$	S_y^2
1	59	61	60	2
2	74	70	72	8
3	50	58	54	32
4	69	67	68	2
5	50	54	52	8
6	81	85	83	8
7	46	44	45	2
8	79	81	80	2

Let's assume σ_y^2 is constant for all the tests.

Calculate pooled sample variance estimate of σ_y^2

$$S_P^2 = \frac{\Sigma s_P^2}{8} = \frac{2 + 8 + 32 + 2 + 8 + 8 + 2 + 2}{8} = 8$$

Full Factorial Example Steve Brainerd

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Summary of Effects: So Which ones are important??
- Look at ***Confidence intervals*** (If 0 is in interval then effect is not significant.)
- $H_o : \mu_{E1} = \mu_{E2} = \mu_{E3} = \ldots = 0$

Data	
72	60
54	68
80	45
52	83
Grand Mean:	64.25

$$s^2_{avg} = \frac{S^2_p}{N} = \frac{8}{16} = 0.5$$

$$s_{avg} = 0.707$$

$100(1 - \alpha)\%$ Confidence interval for average (95% C.I.) is

$$Avg \pm t_{v, 1-\frac{\alpha}{2}} s_{avg} = 64.25 \pm t_{8, 0.975} s_{avg} = 64.25 \pm (2.306)(0.707)$$

Since 0 is not on C.I., reject H_o that $\mu_{avg} = 0$

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Summary of Effects: So Which ones are important??
- Look at Confidence intervals (If 0 is in interval then effect is not significant.)
- Calculate SE and a t value

For our example,

$$s_{eff}^2 = \frac{4}{N} s_P^2 = \frac{4}{16} \cdot 8 = 2$$

Total number of trials = 16,

$$s_{eff} = 1.414$$

Sometimes called the Std. error of an effect
$100(1 - \alpha)\%$ confidence interval

$$Avg \pm t_{v, 1-\frac{\alpha}{2}} s_{avg}$$

95% confidence interval is

$$E_i \pm (2.306)(1.414)$$

$$E_i \pm 3.26$$

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Summary of Effects: So Which ones are important??
- Look at Confidence intervals (If 0 is in interval then effect is not significant.)

95% confidence interval for each effect

$$E_1 = 23.00 \pm 3.26*$$ $$E_{12} = 1.50 \pm 3.26$$

$$E_2 = -5.00 \pm 3.26*$$ $$E_{13} = 10.00 \pm 3.26*$$

$$E_3 = 1.50 \pm 3.26$$ $$E_{23} = 0 \pm 3.26$$

$$E_{123} = 0.5 \pm 3.26$$

0 does not lie on C.I. for E_1, E_2, & E_{13}.
The calculated (sample) effects E_1, E_2, & E_{13} have arisen from a normal distribution not centered at 0.

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Mathematical model

For 2^3 factorial design pilot plant example we tacitly assume that response can be characterized as:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2$$
$$+ b_{13}x_1x_3 + b_{23}x_2x_3 + b_{123}x_1x_2x_3 + \varepsilon$$

We ran tests and fit this equation to the data:

$$\hat{y} = \hat{b}_0 + \hat{b}_1x_1 + \hat{b}_2x_2 + \hat{b}_3x_3 + \hat{b}_{12}x_1x_2$$
$$+ \hat{b}_{13}x_1x_3 + \hat{b}_{23}x_2x_3 + \hat{b}_{123}x_1x_2x_3$$

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Fitted Mathematical model ***Use only A, B, and AC effects in Model!***

We now know that only $\hat{b}_0$, $\hat{b}_1$, $\hat{b}_2$ & $\hat{b}_{13}$ are important.
The fitted model becomes:

$$\hat{y} = \hat{b}_0 + \hat{b}_1 x_1 + \hat{b}_2 x_2 + \hat{b}_{13} x_1 x_3$$

$$\hat{y} = Avg + \left(\frac{E_1}{2}\right) x_1 + \left(\frac{E_2}{2}\right) x_2 + \left(\frac{E_{13}}{2}\right) x_1 x_3$$

$$\hat{y} = 64.25 + 11.5\, x_1 - 2.5\, x_2 + 5 x_1 x_3$$

Design of Engineering Experiments
Chapter 6 – Full Factorial Example

- 2^3 Pilot Plant : Response: % Chemical Yield
- Mathematical model

For each unique combination of x_1, x_2, & x_3 a predicted response may be calculated.

$$\hat{y} = 64.25 + 11.5 \,(-1) - 2.5 \,(-1) + 5(-1)\,(-1)$$

Test#	x_1	x_2	x_3	y_{ij}		$\bar{y}_i$	$\hat{y}_i$	Residuals $e_{ij} =$	$y_{ij} - \hat{y}$
1	-	-	-	59	61	60	60.25	-1.25	0.75
2	+	-	-	74	70	72	73.25	0.75	-3.25
3	-	+	-	50	58	54	55.25	-5.25	2.75
4	+	+	-	69	67	68	68.25	0.75	-1.25
5	-	-	+	50	54	52	50.25	-0.25	3.75
6	+	-	+	81	85	83	83.25	-2.25	1.75
7	-	+	+	46	44	45	45.25	0.75	-1.25
8	+	+	+	79	81	80	78.25	0.75	2.75

Factorial Example

- If there are a levels of Factor A , b levels of Factor B, and c levels of factor C a **_Full Factorial_** design is one in all abc combinations are tested.
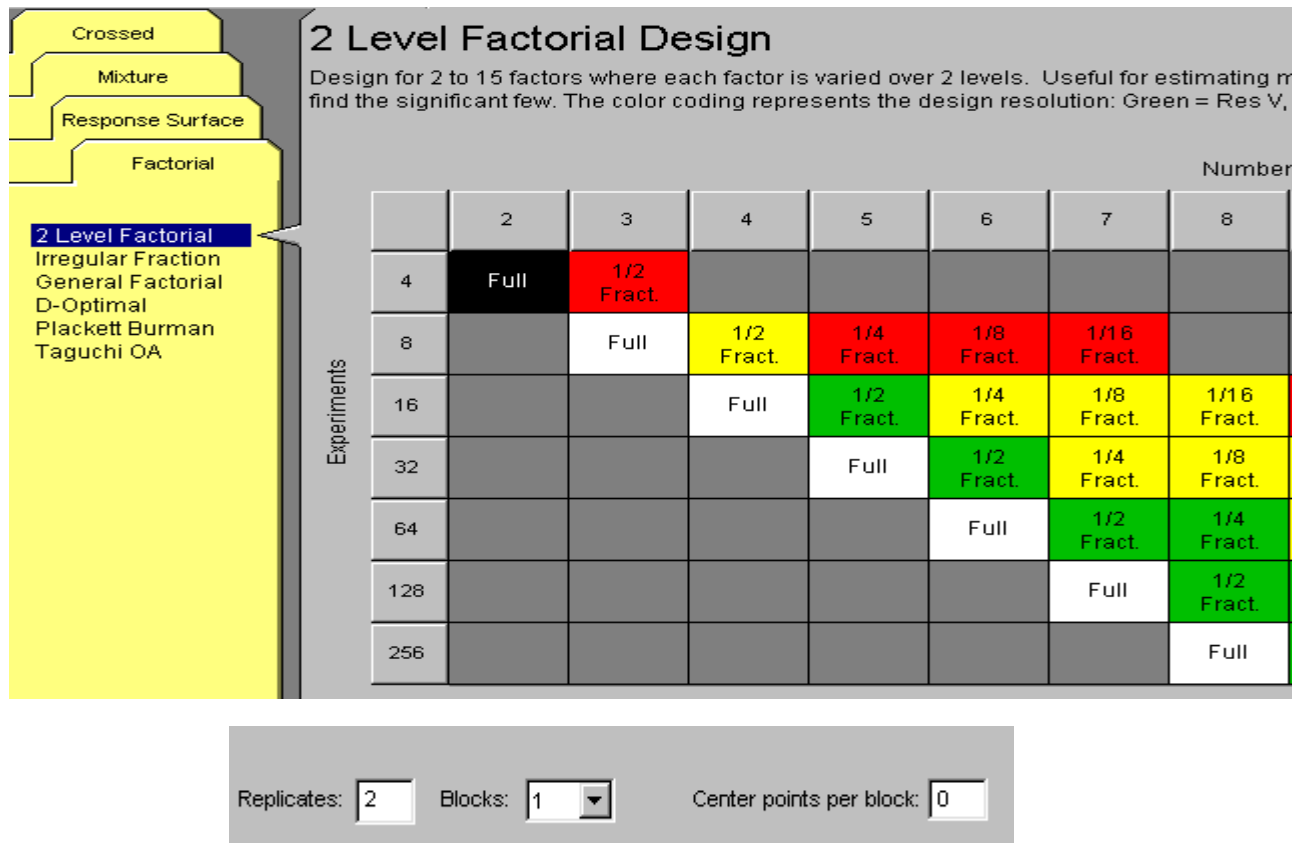
Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example

2 Level Factorial Design

Design for 2 to 15 factors where each factor is varied over 2 levels. Useful find the significant few. The color coding represents the design resolution:

Factors

	Name	Units	Type	Low	High
A:	Temprature	C	Numeric	160	180
B:	Concentration	%	Numeric	10	40
C:	Catalyst	Type	Categorical	A	B

2 Level Factorial Design

Design for 2 to 15 factors where each fact find the significant few. The color coding r

Responses: 1

	Name	Units
	Yield	%

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example

| Std	Run	Block	Factor 1 A:Temprature C	Factor 2 B:Concentration %	Factor 3 C:Catalyst Type	Response 1 Yield %
1	16	Block 1	160.00	10.00	A	59
2	11	Block 1	160.00	10.00	A	61
3	2	Block 1	180.00	10.00	A	74
4	13	Block 1	180.00	10.00	A	70
5	9	Block 1	160.00	40.00	A	50
6	7	Block 1	160.00	40.00	A	58
7	4	Block 1	180.00	40.00	A	69
8	3	Block 1	180.00	40.00	A	67
9	5	Block 1	160.00	10.00	B	50
10	6	Block 1	160.00	10.00	B	54
11	12	Block 1	180.00	10.00	B	81
12	8	Block 1	180.00	10.00	B	85
13	10	Block 1	160.00	40.00	B	46
14	1	Block 1	160.00	40.00	B	44
15	14	Block 1	180.00	40.00	B	79
16	15	Block 1	180.00	40.00	B	81

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: **Half Normal Plot**: Tells you which factors have a significant effect on yield! *Only A, B ,and AC are significant*

Transform	Effects	ANOVA	Diagnostics	Model Graphs

DESIGN-EXPERT Plot
Yield

A: Temperature
B: Concentration
C: Catalyst
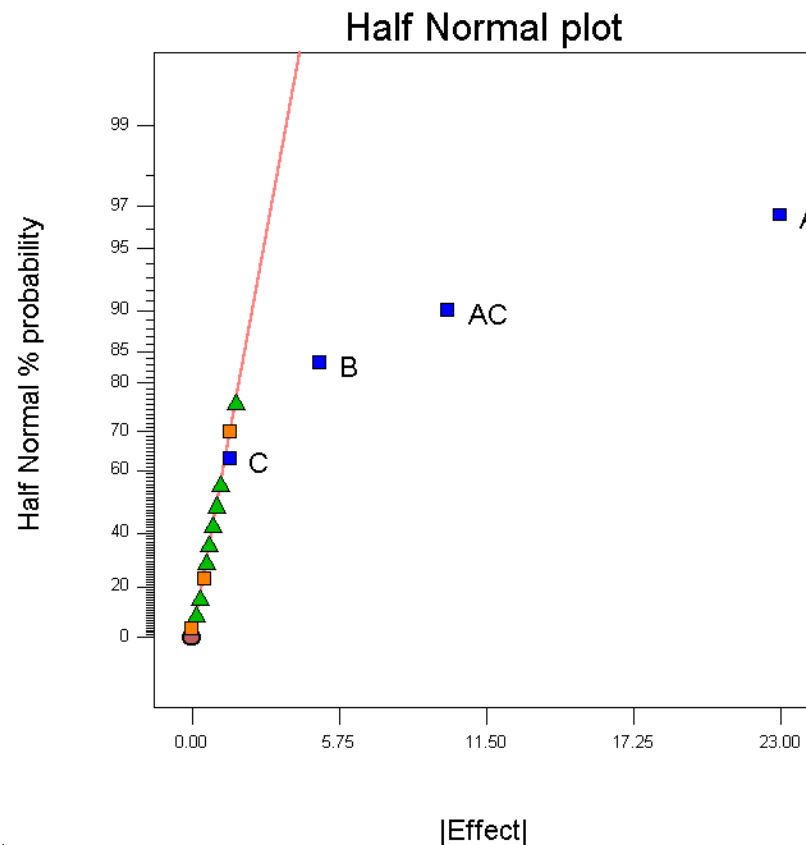
Half Normal plot

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: **Normal Plot:** Tells you which factors have a significant effect on yield! With impact direction. *Only A, B ,and AC are significant*

| Transform | Effects | ANOVA | Diagnostics | Model Graphs |

DESIGN-EXPERT Plot
Yield

A: Temperature
B: Concentration
C: Catalyst

Normal plot
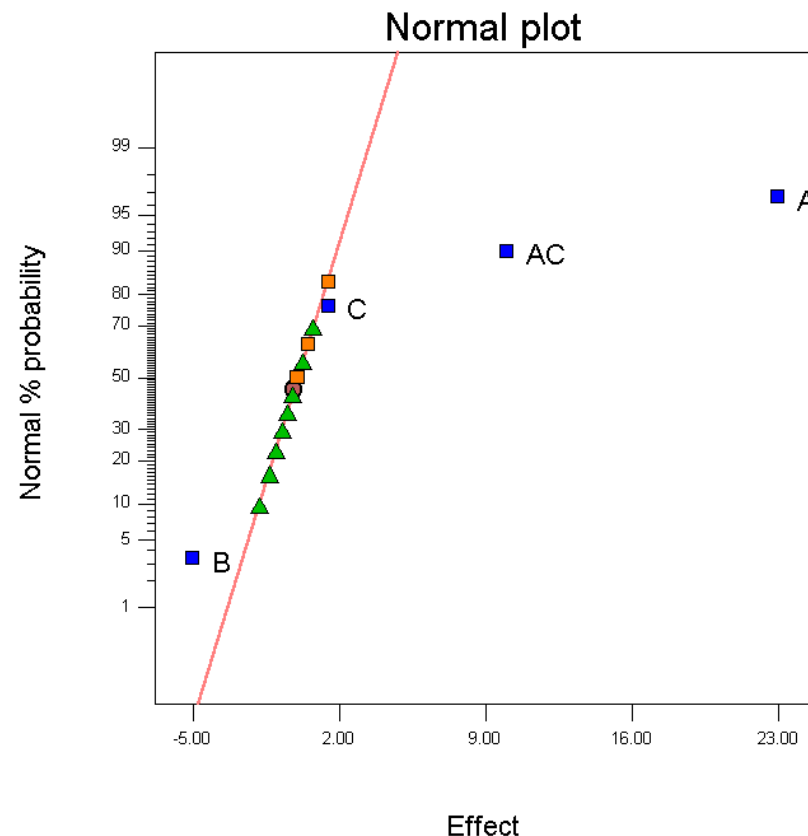
Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: ANOVA Table *Only A, B ,and AC are significant*

Response: Yield

Hierarchical Terms Added after Manual Regression

C

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	2625.00	4	656.25	97.55	< 0.0001
A	2116.00	1	2116.00	314.54	< 0.0001
B	100.00	1	100.00	14.86	0.0027
C	9.00	1	9.00	1.34	0.2719
AC	400.00	1	400.00	59.46	< 0.0001
Residual	74.00	11	6.73		
Lack of Fit	10.00	3	3.33	0.42	0.7459
Pure Error	64.00	8	8.00		
Cor Total	2699.00	15			

Std. Dev.	2.59	R-Squared	0.9726
Mean	64.25	Adj R-Squared	0.9626
C.V.	4.04	Pred R-Squared	0.9420
PRESS	156.56	Adeq Precision	26.208

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: ANOVA Table ***Only A, B ,and AC are significant***

Factor	Coefficient Estimate	DF	Standard Error	95% CI Low	95% CI High	VIF
Intercept	64.25	1	0.65	62.82	65.68	
A-Temperature	11.50	1	0.65	10.07	12.93	1.00
B-Concentration	-2.50	1	0.65	-3.93	-1.07	1.00
C-Catalyst	0.75	1	0.65	-0.68	2.18	1.00
AC	5.00	1	0.65	3.57	6.43	1.00

Final Equation in Terms of Coded Factors:

Yield =

+64.25

+11.50 * A

-2.50 * B

+0.75 * C

+5.00 * A * C

So Why is C also used in the modeled equation if it is not significant?

There is a rule called the ***hierarchy principle***!
See page 203

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: ANOVA Table ***Models***

Final Equation in Terms of Actual Factors:

Catalyst   A

Yield =

-42.83333

+0.65000   * Temperature

-0.16667   * Concentration

Catalyst   B

Yield =

-211.33333

+1.65000   * Temperature

-0.16667   * Concentration

So Why are there 2 equations listed?
Why isn't it just one equation with all terms?

See page 203

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: ANOVA Table ***Only A, B ,and AC are significant***

Change all Factors to Quantitative

Std	Run	Block	Factor 1 A:Temperature C	Factor 2 B:Concentration %	Factor 3 C:Catalyst Type	Response 1 Yield %
14	1	Block 1	160.00	40.00	1.00	44
3	2	Block 1	180.00	10.00	0.00	74
8	3	Block 1	180.00	40.00	0.00	67
7	4	Block 1	180.00	40.00	0.00	69
9	5	Block 1	160.00	10.00	1.00	50
10	6	Block 1	160.00	10.00	1.00	54
6	7	Block 1	160.00	40.00	0.00	58
12	8	Block 1	180.00	10.00	1.00	85
5	9	Block 1	160.00	40.00	0.00	50
13	10	Block 1	160.00	40.00	1.00	46
2	11	Block 1	160.00	10.00	0.00	61
11	12	Block 1	180.00	10.00	1.00	81
4	13	Block 1	180.00	10.00	0.00	70
15	14	Block 1	180.00	40.00	1.00	79
16	15	Block 1	180.00	40.00	1.00	81
1	16	Block 1	160.00	10.00	0.00	59

Full Factorial Example Steve Brainerd

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert
  Example: ANOVA Table ***Use A, B ,C, and AC in Model***

Final Equation in Terms of Coded Factors:

Yield =

+64.25

+11.50 * A

-2.50 * B

+0.75 * C

+5.00 * A * C

Final Equation in Terms of Actual Factors:

Yield =

-42.83333

+0.65000 * Temperature

-0.16667 * Concentration

-168.50000 * Catalyst

+1.00000 * Temperature * Catalyst

Change all Factors
to Quantitative

Final Model:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3$$

$$+ b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3$$

$$+ b_{123}x_1x_2x_3 + \varepsilon$$

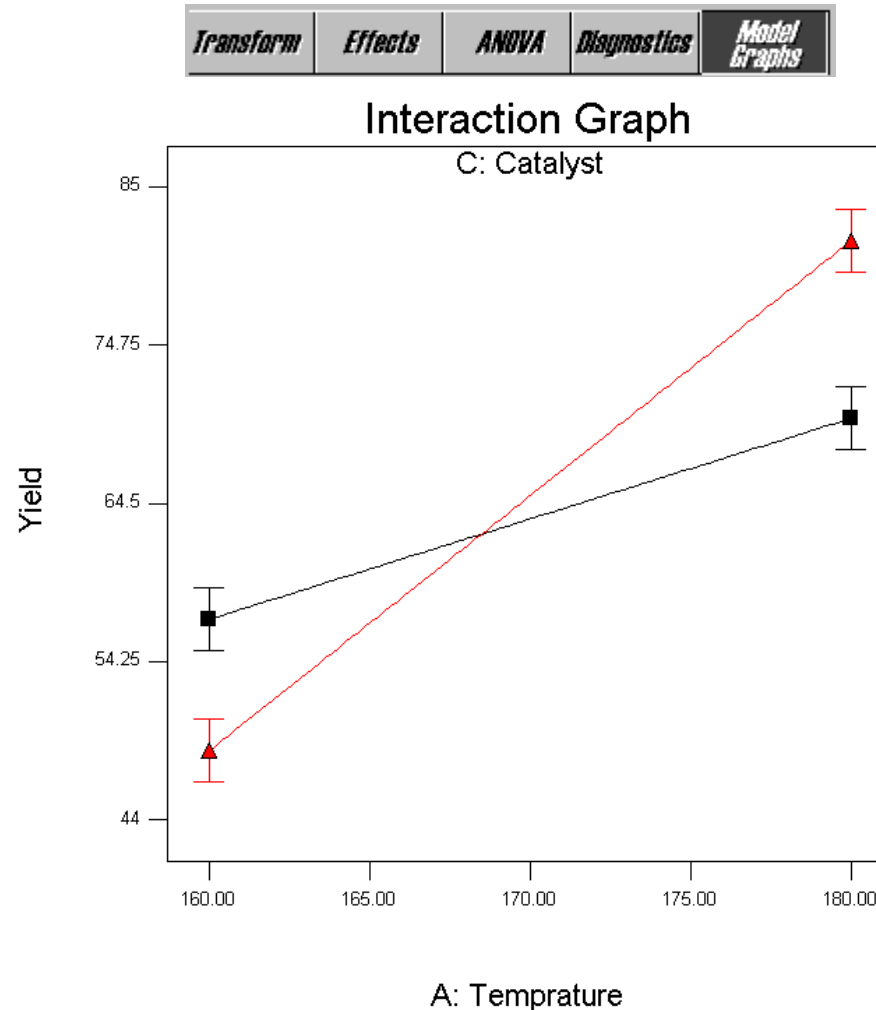Full Factorial Example Steve
Brainerd

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: Residuals Plot and run sequence: What do these plots tell us??
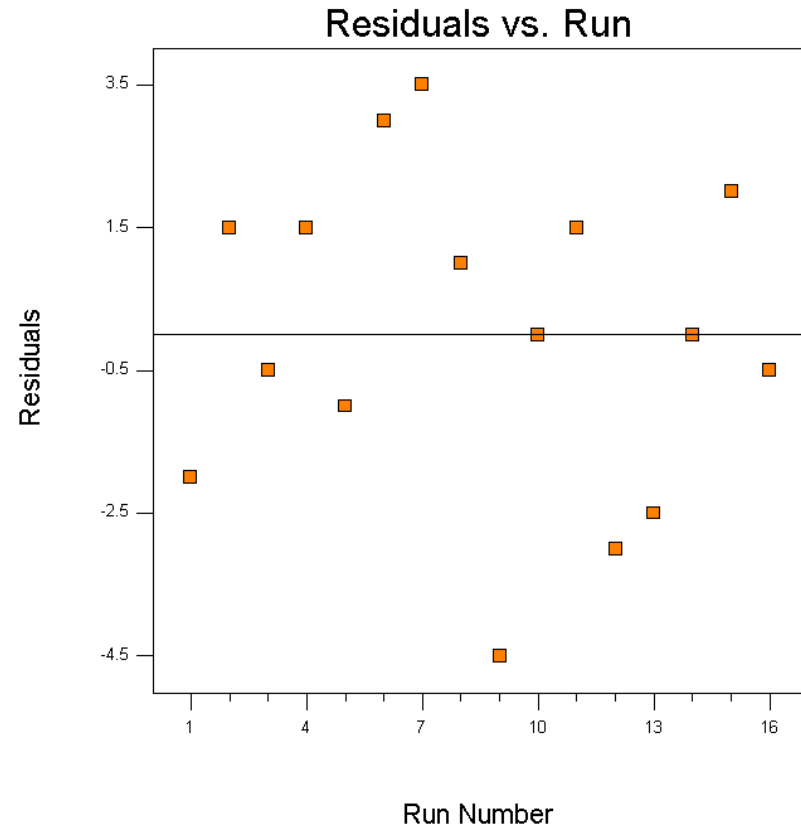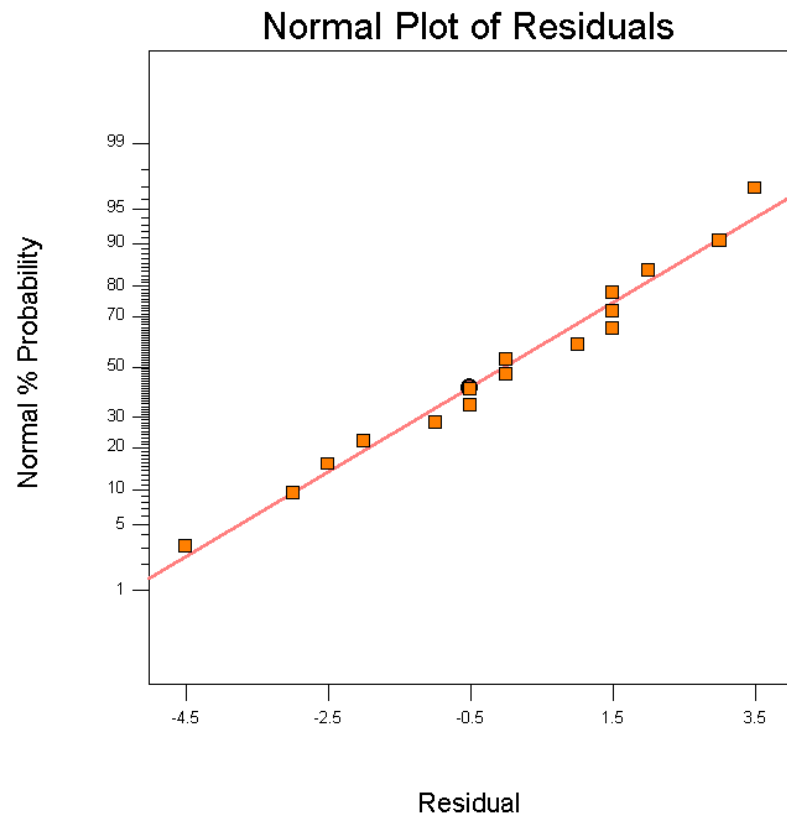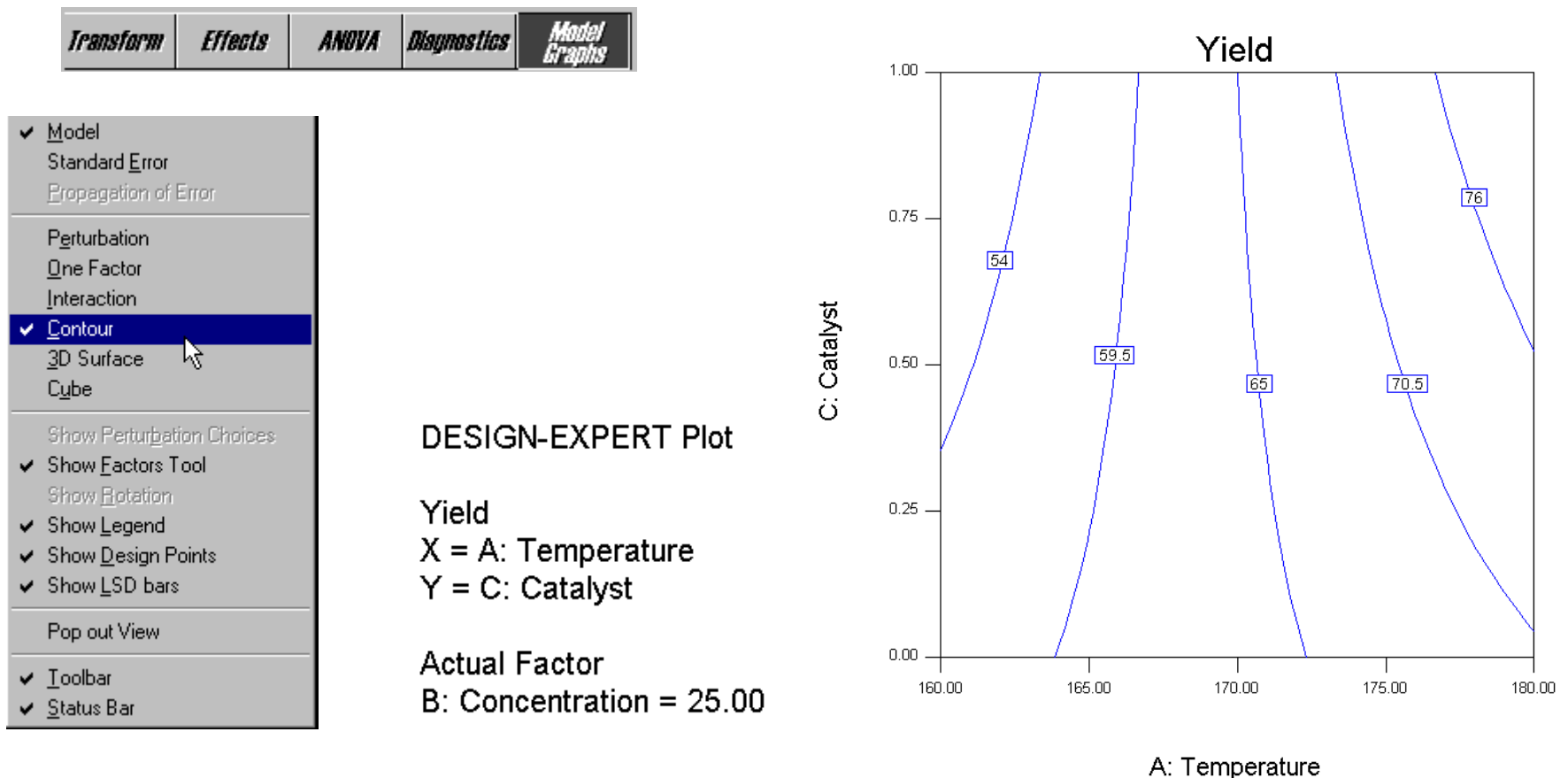
Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: Model Graphics: Interaction Plot *Only A, B , C, and AC used*
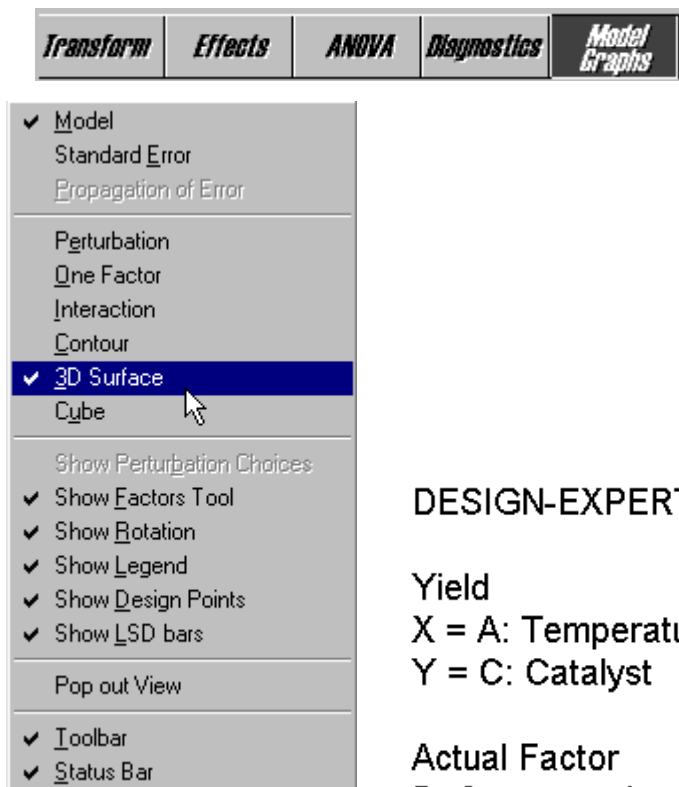
Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: Model Graphics: Interaction Plot ***Model Contour: Catalyst Average Only A, B ,C, and AC used in Model***

DESIGN-EXPERT Plot

Yield
X = A: Temperature
Y = C: Catalyst

Actual Factor
B: Concentration = 25.00

Factorial Designs

- 2^3 Pilot Plant : Response: % Chemical Yield Design Expert Example: Model Graphics: Interaction Plot *Model 3D Surface: Catalyst Average Only A, B ,C, and AC used in Model*

| Transform | Effects | ANOVA | Diagnostics | Model Graphs |

✓ Model
Standard Error
Propagation of Error

Perturbation
One Factor
Interaction
Contour
✓ 3D Surface
Cube

Show Perturbation Choices
✓ Show Factors Tool
✓ Show Rotation
✓ Show Legend
✓ Show Design Points
✓ Show LSD bars

Pop out View

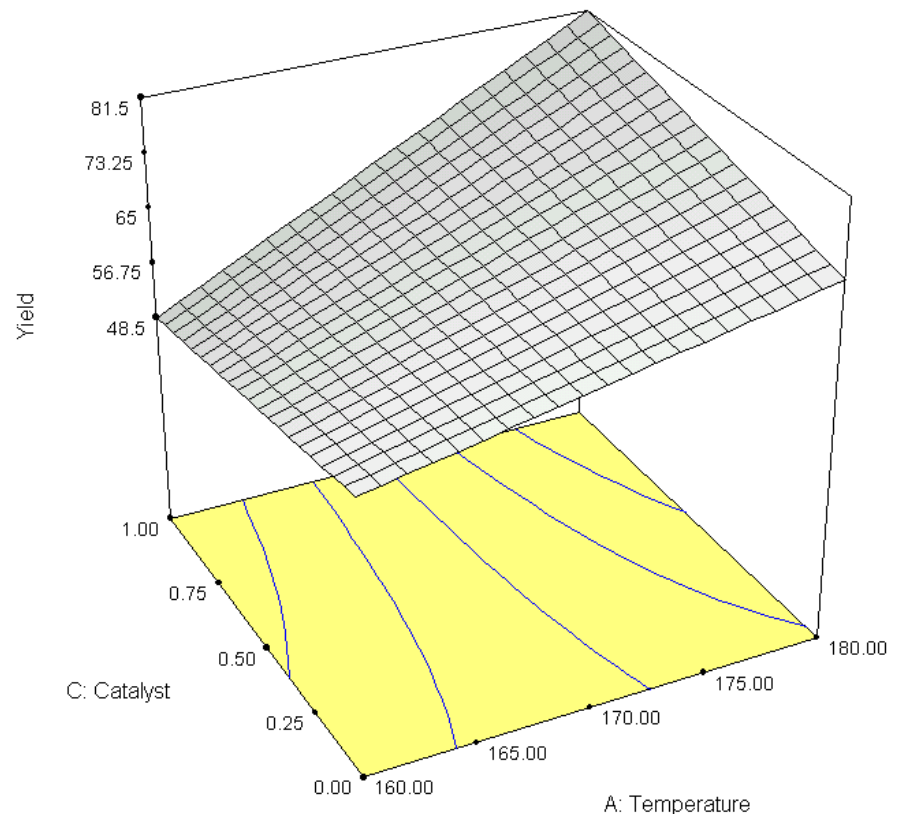✓ Toolbar
✓ Status Bar

DESIGN-EXPERT Plot

Yield
X = A: Temperature
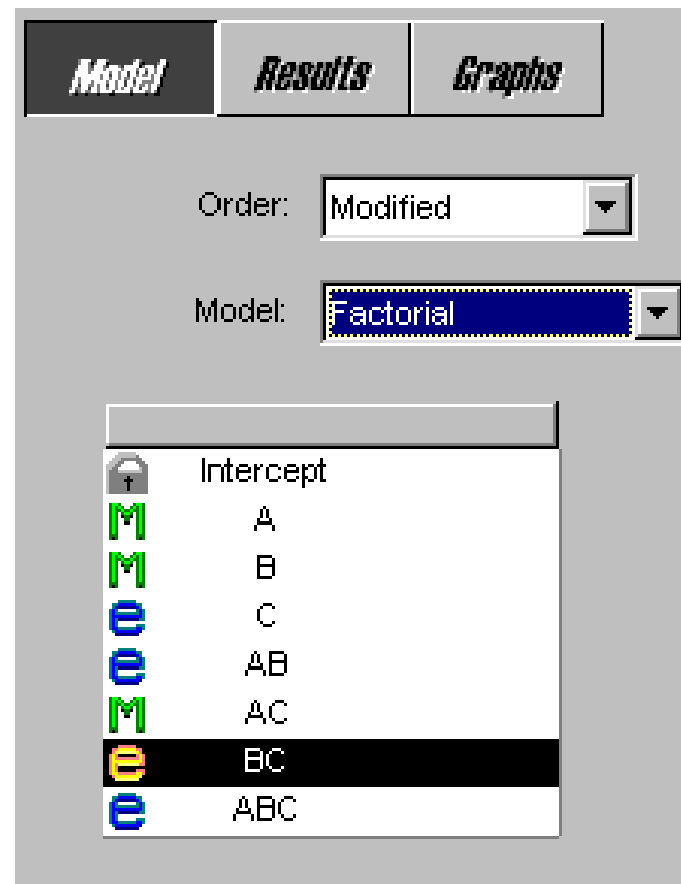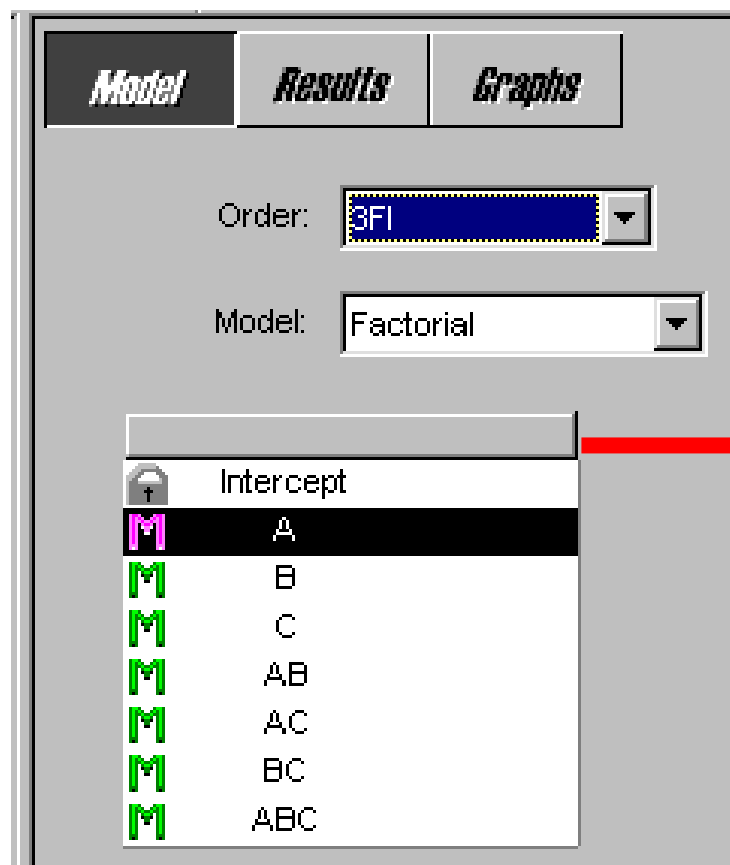Y = C: Catalyst

Actual Factor
B: Concentration = 25.00

Factorial Design: Design Expert

Model terms: 3FI = factor interaction

• Only A, B ,and AC are significant So Change Model>> Modified

Factorial Design: Design Expert

Model terms: 3FI = factor interaction

• *Only A, B ,and AC are significant Change Model>>*

Response: Yield

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	2616.00	3	872.00	126.07	< 0.0001
A	2116.00	1	2116.00	305.93	< 0.0001
B	100.00	1	100.00	14.46	0.0025
AC	400.00	1	400.00	57.83	< 0.0001
Residual	83.00	12	6.92		
Lack of Fit	19.00	4	4.75	0.59	0.6772
Pure Error	64.00	8	8.00		
Cor Total	2699.00	15			

Std. Dev.	2.63		R-Squared	0.9692
Mean	64.25		Adj R-Squared	0.9616
C.V.	4.09		Pred R-Squared	0.9453
PRESS	147.56		Adeq Precision	28.898

Normal plot

Full Factorial Example Steve Brainerd

45
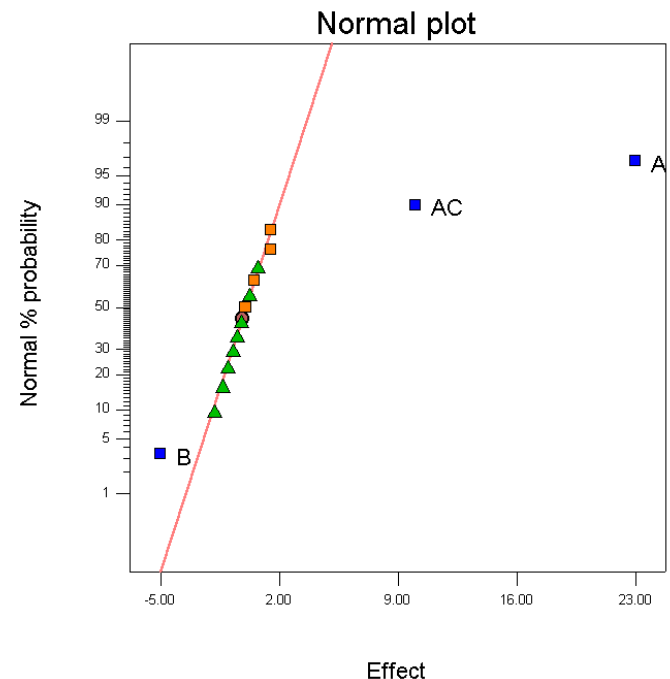
Factorial Design: Design Expert

Model terms: 3FI = factor interaction

• *Only A, B ,and AC used in Model*

Factor	Coefficient Estimate	DF	Standard Error	95% CI Low	95% CI High	VIF
Intercept	64.25	1	0.66	62.82	65.68	
A-Temperature	11.50	1	0.66	10.07	12.93	1.00
B-Concentration	-2.50	1	0.66	-3.93	-1.07	1.00
AC	5.00	1	0.66	3.57	6.43	1.00

Final Equation in Terms of Coded Factors:

Yield =
+64.25
+11.50 * A
-2.50 * B
+5.00 * A * C

Final Equation in Terms of Actual Factors:

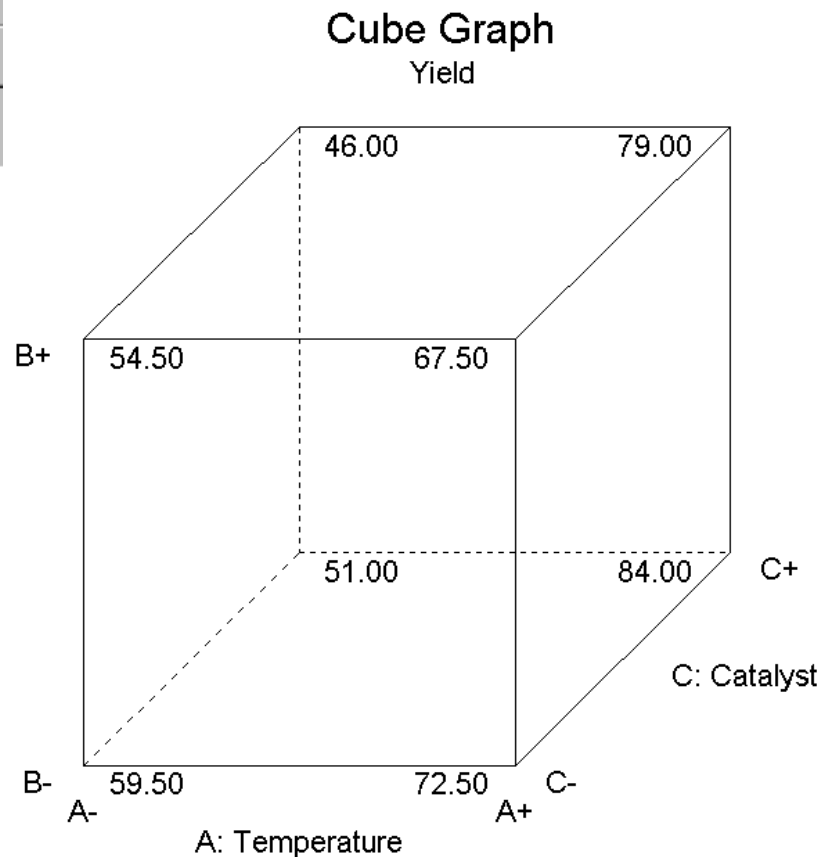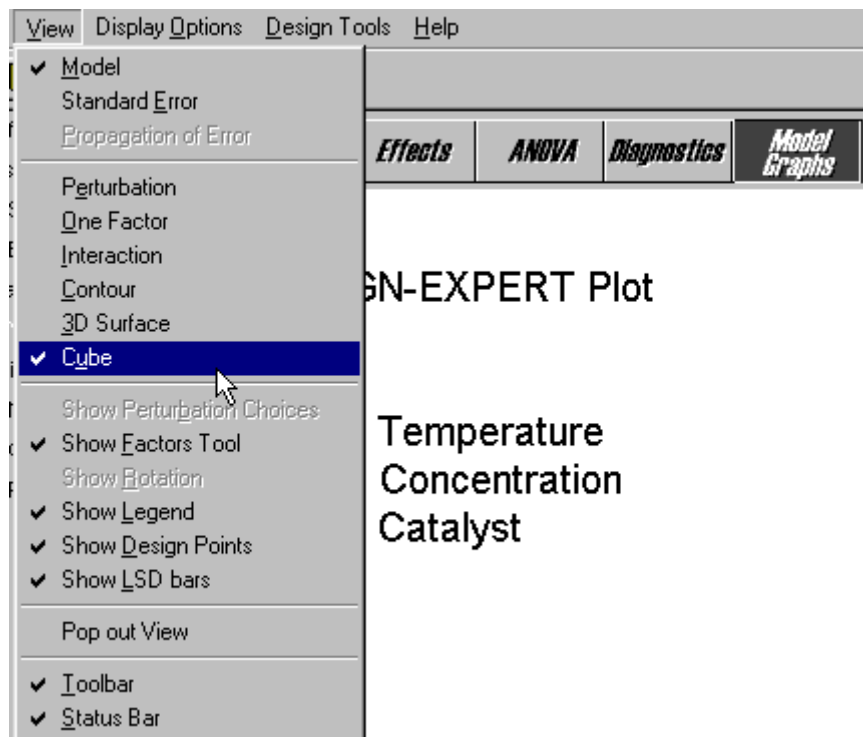Not available, because this model is not hierarchical.

Only hierarchical models are scale independent and can be translated into actual units.

Full Factorial Example Steve Brainerd

Factorial Design: Design Expert
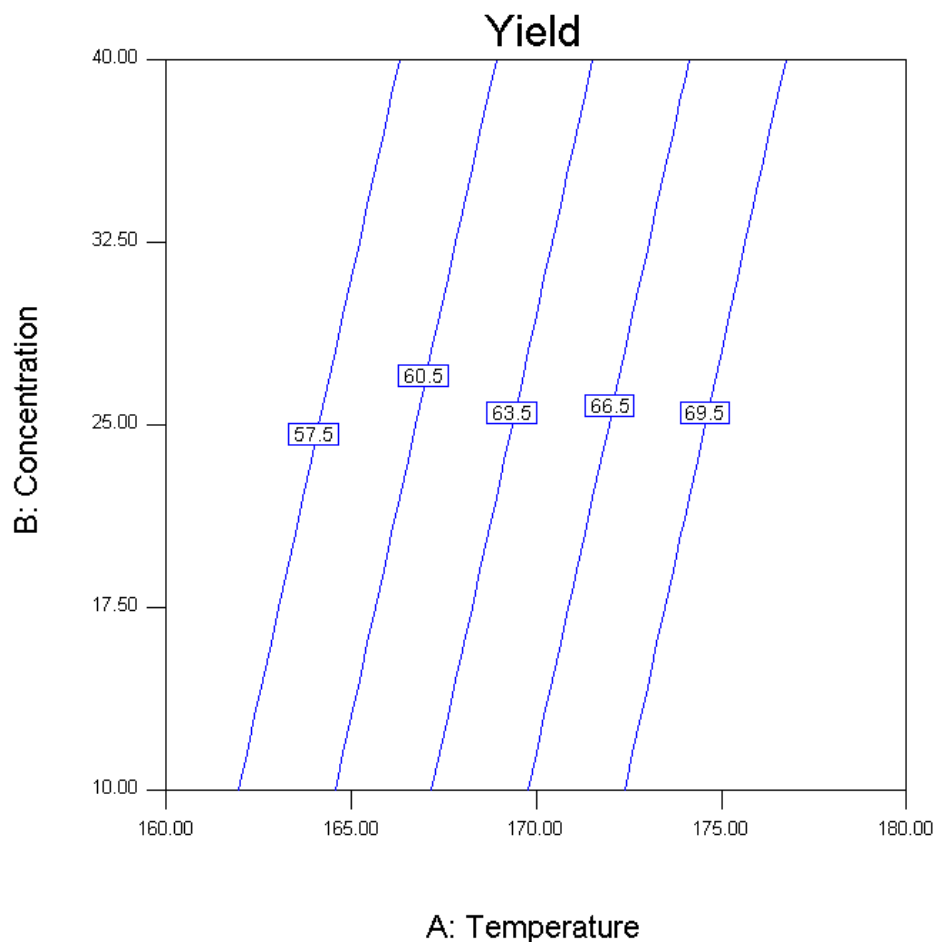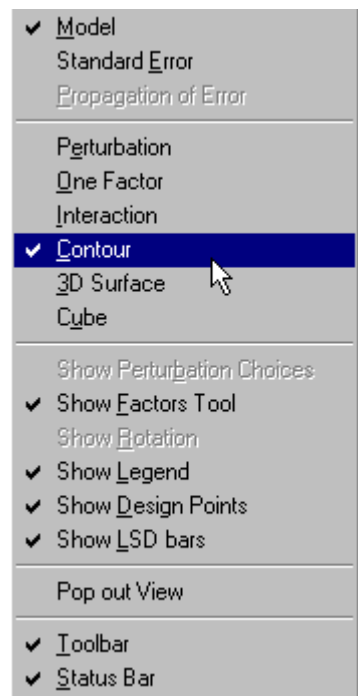
View: Modeled data on cube layout

- *Model Cube: Only A, B ,and AC used in Model*

Factorial Design: Design Expert

•*Model Contour: Catalyst Average Only A, B ,and AC used in Model*

Full Factorial Example Steve Brainerd

Factorial Design: Design Expert

View:

•Model 3D Surface Catalyst Average : Only A, B ,and AC used in Model

Full Factorial Example Steve Brainerd